OPERATIONS RESEARCH AND THE ACCOUNTANT

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ABSTRACT

There has been a certain amount of scepticism and even apprehension on the part of accountants with respect to the sudden appearance of scientists in the affairs of the business world. A particularly perplexing feature of operations research is the use of mathematical language, even when dealing with conventional problems. This paper explains in layman's language, why scientists use mathematics both in the physical sciences and in the problems of the business world. The businessman of the future need not become a mathematician, as mathematics in business may be considered as a highly sharpened form of common sense. As an illustration of the use of mathematics, a cost accounting problem is discussed and it is shown that a statistical definition of "overhead" can lead to simplified pricing methods and management controls. Also, some of the confusing aspects of "overhead" accounting can be avoided by using mathematical techniques.

OPERATIONS RESEARCH AND THE ACCOUNTANT

Andrew Vazsonyi

Perhaps the best way to begin a discussion on operations research is to give a definition of what operations research is. This sets the stage in a scholarly fashion, and then the speaker can develop his theme. The difficulty, however, is that different operations researchers give different definitions of operations research and it is hard to find two operations researchers who agree on the same definition. To demonstrate this point, here is a list of a number of definitions that have been given:

Definition 1: Operations research is the science of decision.

Definition 2: Operations research is the application of the methods of physical science to provide quantitative answers to executives, with regard to operations under their control.

Definition 3: Operations research is quantitative common sense.

Definition 4: Operations research is what operations research workers do.

Definition 5: Operations research is what operations research workers *think* they do.

Definition 6: Operations research is simultaneously industrial engineering, statistics, quality control, market analysis, cost accounting, civil engineering, applied mathematics, applied psychology, and econometrics.

Definition 7: The fact of the matter is that operations research is none of these.

Definition 8: Operations research is at present undefined, but in time will become defined by the subject matter appearing in the Journal of the Operations Research Society of America.

At this point, I could add my own personal definition of operations research, but I do not believe this would clear matters up. In fact, I suggest that we should not concern ourselves about what operations research is, and should give up trying to give a definition. You might say that this is a very unsatisfactory state of affairs, as how can I talk about a subject that I cannot define. To tell you the truth, I always felt that cost accounting is somewhat of a mysterious subject, and some time ago I decided to find out what cost accounting is. Obviously, cost accounting refers to costs, so I became curious to know what is really meant by the word "cost" in cost accounting. I was fortunate to find an appropriate paragraph in the Cost Accountants Handbook, Section 5, on page 217. I shall proceed to quote this illuminating definition of the word "cost."

> "The word cost in an accounting sense cannot be defined unconditionally. Cost becomes an individual formula in each business enterprise. Cost to some means the actual money outlay, past and present, for the cost of production.......to others, cost includes not only the cost of production, but, in addition, the marketing and administrative expense combined to represent an over-all or commercial cost. The varied nature of production and the size of the business has something to do with its formulation."

I cannot define operations research, you cannot define cost. However, I am sure we can talk about some distinctive characteristics of our respective fields. Therefore, I propose to describe some of the unique features of operations research.

One of these features can perhaps be illustrated best through a little episode. A few days ago, I was talking to a vice president and he told me that he has been playing golf with a certain Dr. Clark. Recently, he tells me, he got pretty well acquainted with Dr. Clark in the bar, and he asked Dr. Clark, "Doc, where is your office?" Dr. Clark said that he didn't have an office because he is not an M.D., he is, in fact, chief of Market Research for a certain corporation. He is a Ph.D who is involved in the solution of some difficult business problems.

The point I wish to make here is that during recent years there has been a steady increase in the number of scientists involved in business affairs. You may ask me, is this something really new? I think, yes! The traditional scholar stays within his ivory tower and does not mingle with the business world. In fact, not long ago, it would have been considered shameful for a scientist to be engaged in affairs of real life. It was said about Archimedes, the classic Greek scientist who lived about a couple of thousand years ago, that "although practical inventions had obtained for him the reputation of more than human sagacity, he did not deign to leave behind any written word on practical subjects but regarding as ignoble and sordid the business of mechanics and every sort of art which is directed to use and profit, he placed his whole ambition in those speculations, the beauty and subtlety of which are untainted by any admixture of the common weed of life."

I think these words describe with great accuracy the attitude of the classic scholar. During World War II some of our best scientists got involved in such mundane affairs as radar and the atomic bomb. After the end of World War II, it occurred to a number of scientists that perhaps they could make contributions to some even more mundane subjects, such as inventory control, market research, and so on. A sudden influx of scientists into the affairs of the business world followed, first under the label of operations research and more recently, under the name of management sciences. I want to remind you though, that not all scientists in business are listed under these new titles.

This is the first point then that I wish to make, that scientists are taking an important role in business. Now, scientists are peculiar people, and sometimes the methods they use in solving problems are peculiar too. Let us, for the moment, imagine that we take the Journal of the Operations Research Society of America, or we take Management Science, and we examine, in a superficial way, the papers published there. Both of these journals are dedicated to solving business problems. Even a casual observer would be struck by some peculiar symbols in these journals. He would be amazed and disturbed to find that both of these journals are full of mathematical equations. Is all this mathematics necessary? Granted that scientists use mathematical in their own fields, but why do they try to impose mathematics on the business world?

Here, if I may be permitted to digress for the moment, I would like to say a few words about what I call the curious attitude of laymen towards mathematics. My initial discussion with a business executive, and his first reaction may be paraphrased as follows. "Doc, I want to warn you that I know nothing about mathematics. I had no particular trouble with math in kindergarten, I did fairly well even in elementary school, but when it got to high school things were way over my head." At this point, he may smile, lean back on his upholstered chair and put on a well-satisfied air. He is indeed very proud of being ignorant about mathematics. Now, when it comes to golf, he is not so proud to admit he shoots a hundred. He does not even like to admit that he is a very poor poker player. However, when it comes to mathematics he feels that it is admirable to be ignorant of mathematics. Why is this?

As far as the physical scientists are concerned, mathematics is a concise language which allows the description of certain things better than is possible with only words. Mathematics is a sharpened common sense, and in most scientific problems the language of mathematics is the appropriate one. Now, we are finding out that in some business problems, mathematics is the appropriate language. You might tell me, at this point, that business cannot be run by mathematics and, in particular, business cannot be run by formulas. This is not what I mean. In order to get across the idea of what I mean, I will proceed to describe first some problems from the physical sciences and explain how mathematics is used there. Then I will talk about a cost accounting problem and suggest some ways in which mathematics may be helpful.

The first problem I want to consider is from physics; it is the problem of the falling stone. Suppose we make a lot of experiments and find out how far a stone falls in a given time. We find, for instance, that in one second a stone falls 16.1 feet; that in three seconds a stone falls 1450 feet. We take all of the various observations and put them into a table, somewhat like this:

Seconds	Feet
1	16.1
2	64.5
3	145.0
1/2	4.0
1/3	1.8
1/5	.6

Now, we could take thousands and thousands of observations, and make up books full of tables telling us how far the stone would fall. This would be a way of presenting the information on the law of free-fall. The first disadvantage of this presentation is that we need books and books to answer our questions. But even all these tables are not sufficient because one cannot list all possible distances and times that a stone might fall. In spite of the fact that we might have a library of books, the information still would be incomplete. The mathematical way to describe this same phenomena is to write:

$$D = 16.1t^2$$

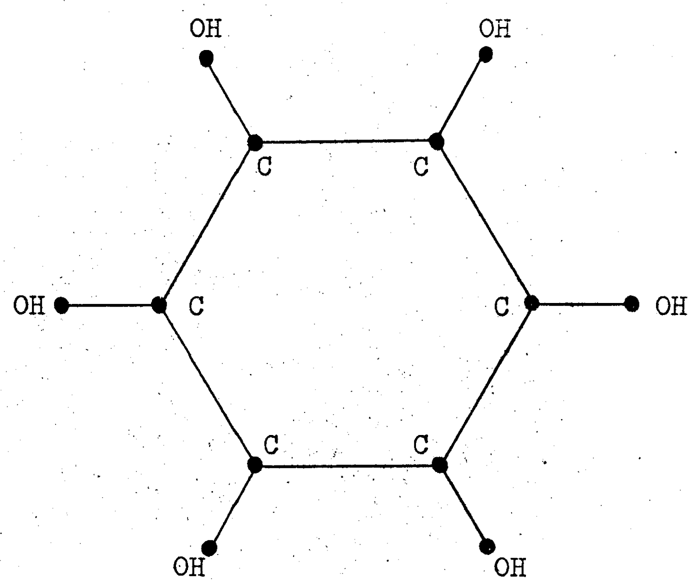
This is a mathematical equation where D denotes the distance that the stone falls, and t refers to the time of fall in seconds. We can substitute in any value of t, say 2 seconds, 3 seconds, or 1/2 second; we compute the square of this number as indicated by the equation, multiply by 16.1, and then get the distance that the stone would fall. Compare the effectiveness and conciseness of our mathematical equations to the hypothetical library containing our tables! We have a single equation which is rather easy to remember. Furthermore, we can compute the distance of fall for any given time. For those people, then, who understand the equation, it is obvious that the equation is much superior to the library of tables. This looks very simple. However, it took thousands of years to develop this particular equation. Even if the law of free-fall had been known a few hundred years ago, the equation itself would have been written in a much more cumbersome and confusing fashion. What appears today as a very simple matter, represents thousands of years of progress of scientific thought. To those of you who are not used to mathematical equations, this equation may appear strange. However, I believe it is not hard to understand what the equation stands for, and you must agree that this is, indeed, a better way of describing the problem of free-fall.

Now, we can go one step further with the same problem. Suppose I ask the question, how long is it going to take for a stone to fall 100 feet? If we had a library of tables we could look up the distance, 100 feet, and if we are lucky we would find the distance 100 and read the time of fall as 2.5 seconds. (If we are not lucky, we may not find the distance of 100 feet in our tables.) However, we can take our equation of free-fall and to use the language of the mathematician, we can "solve" it for time t, and write

$$t = .25\sqrt{D}$$

This is, then, the equation that tells us how long it takes for a stone to fall the distance D. The funny sign on the right-hand side is the so-called square root sign. (It is a distorted r referring to the Latin word <u>radicalis</u> which stands for root.) The square root of 100 is 10 and, therefore, the formula tells us that it will take .25 times 10 seconds, that is, 2.5 seconds for the stone to fall 100 feet. Again, I want to emphasize that the development of such a simple formula took thousands of years of scientific thought. However, once the symbols in the equations are understood, we have a good way of describing the solution to the problem of free-fall.

Most laymen think that mathematics deals exclusively with numbers and formulas. This is not so. To give you an illustration, I ask those of you who studied chemistry to recall the organic compound, benzene. Some properties of this compound are described with the aid of the diagram below:



Now, to the student of chemistry, this diagram means a great deal. It tells them how many carbon, how many oxygen, and how many hydrogen atoms are in a molecule of benzene. It tells them a great deal more because the diagram refers to the structure of the benzene molecule. I challenge anyone to describe this structural property of the benzene molecule without the diagram shown here.

One could replace the diagram with a verbal description but this verbal description would be so confusing that it would probably be useless. The best way to represent this property of the benzene molecule is to use this diagram. Here again, we have an example where mathematics (or perhaps, more accurately, geometry), describes some phenomenon in a better way than any verbal description can hope for.

Before I proceed to tell you how the language of mathematics can be used in a business problem, I would like to make a further point in connection with these illustrations from the physical sciences. Namely, I anticipate the objection that business cannot be described with the aid of formulas. One might say the very same thing about physical problems. I told you that the fall of a stone can be described with an equation. Is this really true? Some people might say that I have, in fact, misled you. To begin with, what about air resistance? The friction of air influences the fall of bodies and I have omitted this from my equation. What about the particular shape and roughness of the body? These I have left out of the equation, and therefore my equation is in error. Now, it is possible to write more complex equations that will include some of these more complicated aspects of the problem of free-fall. However, no equation will ever completely describe all these ramifications. Fortunately, in most cases, we are not really interested in these various complicating factors.

I have talked about the diagram related to the benzene molecule. I said this described the structure of the benzene molecule. Is this really true? What about the odor of benzene? What about the consistency? Is it poisonous? Would it burn? All these things I have left out. However, this does not mean that this representation of the benzene molecule is useless. It describes only some properties of benzene and not all of them.

When we describe a phenomenon with a mathematical equation, we always idealize the problem. We describe the motion of a hypothetical stone which falls in a vacuum. This idealized description abstracts some basic and important features of the phenomenon, and expresses this basic feature in a general sort of fashion. This is the same sort of a thing that we do in connection with a business problem. We idealize the problem, abstract some basic aspects and describe this in a mathematical way. It is important in this work to have a strong sense of the realities of the business, so that we indeed abstract the essentials. It is also important that at least at the beginning, we concentrate on some particular features of the business problem, and we do not attempt to describe the problem with all its ramifications. This is similar to our method of describing the motion of the falling stone, when, at the beginning, we concentrate on free-fall in vacuum. When we obtain a mathematical description of a business problem, we have to keep in mind that we have described only some particular features of the problem and we should not expect our mathematics to give answers to those aspects of the business problem that we have not included in our mathematics.

I am afraid, at this point, that this discussion is beginning to become too abstract and philosophical. I think the best way now to proceed is to show you an illustration of how I think mathematics could be used in cost accounting. I will follow my own recipe and consider a hypothetical firm, concentrating only on some limited aspects of the problem of cost accounting.

Cost Accounting in The Alpha Corporation

The Alpha Corporation is in business to sell services. We do not spell out the details here, of what these services are, but as an illustration you may imagine a management consulting firm as a corporation that sells services.

The Alpha Corporation bills its customers at a flat rate per dollar of "direct" labor. The customer sets up some rules of what labor can be considered "direct," or indirect. For instance, clerical people or supervisors are not considered as direct labor. Also, it is recognized that some of the employees of The Alpha Corporation work on many different contracts, and it is impractical to keep track of their time every minute. Consequently, the corporation follows the rule that only labor performed in excess of half a day is recorded against a contract, and the rest of the labor is accounted for as indirect labor. (The important thing is that there are definite rules in deciding whether or not a man is considered a direct or an indirect charge.) In a more general sense, we could say that the corporation incurs certain allowable expenses, that is, expenses that can be directly charged to the customer, and then some other expenses that are not allowable. In order to be more specific, in Table 1 we show an illustrative record for the year of 1955. This table shows a monthly record of direct labor in dollars, or if you wish, allowable expenses and also it shows indirect or not allowable expense. Now, the management of this Alpha Corporation is relying on these data in evaluating the future course of business. The Market Research Department predicts the level of business for The Alpha Corporation, and the management desires to have a multitude of questions answered. Management would like to predict gross profits, and to know what the profit would be, depending on alternate possible billing rates. They would like to know what the profit is going to be after taxes. They would like to know what dividends they can pay to stockholders. As there are no inventories in The Alpha Corporation, they do not need to worry about inventory carrying costs or obsolescence. As there is no substantial equipment here, there is no need to study investments, depreciation, capital gains, and many other things.

In a real-life situation, of course, most of these questions and many others would have to be answered. However, as I said before, I am dealing with a highly idealized type of corporation, and I am interested only, for the moment, in answering certain specific questions. First of all, I want to determine the gross profit, assuming that there are alternate forecasts for the level of business. An analyses of past data in Table 1 indicates that "the overhead" decreases as the level of business increases. In order to compute costs and profits, it is necessary to put this decrease of overhead into some sort of a simple rule. Now, here is a place where mathematics may be useful. Figure 1 presents a chart which for a cost accountant may look strange, but for a mathematician or a statistician looks familiar. The horizontal axis is the direct labor in thousands of dollars taken from Table 1, and the vertical axis is the indirect laobr (or the not allowable expense), in thousands of dollars. Each little cross in Figure 1 represents the accounting data from Table 1 for a single month. For instance, Point A represents the month of January, as during this month direct labor was up $24,000 and indirect labor was $16,000. We see then that the accounting data is represented by a group of crosses in Figure 1, and our problem is to develop a summarized mathematical representation of these data. In the language of the statistician we wish to determine the regression line expressing the indirect labor as a function of the direct labor. This is a problem that statisticians have studied for a long time, and techniques have been developed to solve such problems. Without going into details, we may say that the problem is to put a straight line through the crosses in such a way that this straight line goes close to these crosses. When such a straight line is obtained, we can write the "equation" of this straight line in the following form:

$$e = 6000 + \frac{1}{2}L \qquad (1)$$

In this equation, L denotes the direct labor, and e denotes the expense, or the indirect labor. The formula says that in order to get the expense we need to take the direct labor, take half of it and add $6000. For instance, if the direct labor L is $24,000, we need to take half of this which is $12,000 and add $6000 and so we get the indirect labor of $18,000. This is represented in Figure 1 by Point B. We recognize that our formula does not give exactly the expense for the month of January because instead of $16,000 it gives the value $18,000. However, we note from the diagram that when direct labor is $14,000 there may be three different amounts of indirect labor, $16,000, $17,000, and $19,000. Our formula gives only a sort of an average value. We see, then, that our mathematical equation represents an idealized summarization of our data, and as far as forecasts are concerned, this equation may be used to compute future expenses. Let me remind you that this mathematical equation does not give you any information that we do not have in Table 1. In fact, the mathematical equation contains only a part of the information since, for instance, it does not tell you how far the actual indirect labor expenses may vary from this idealized equation. But as far as the future is concerned, we cannot tell when these variations will occur and, therefore, we feel that for forecasts the use of this equation may be adequate.

I say this somewhat with tongue in cheek, because you recognize that there may be doubts in connection with the use of this formula. There is a possibility that indirect expense shows a seasonal variation as, say, people take more vacations in the summer than in the winter. Or, there might be some other unrecognized facts that affect indirect charges. However, for the time being, I disregard these possibilities just the same as I disregarded the objections against my equation solving the problem of the falling stone.

The next thing I want to put into mathematical language is the formula for the total expense involved in The Alpha Corporation. This is obtained by adding the direct labor and the expenses, or in mathematical form

$$C = e + L \tag{2}$$

We can combine this last equation with our first equation and get an alternate expression for the total cost in The Alpha Corporation:

$$C = 6000 + \frac{1}{2}L + L = 6000 + 1.5\,L \tag{3}$$

We can see then that the total cost for the corporation is obtained by taking the direct labor cost, multiplying it by 1.5 and adding $6000. In Figure 2, we show this total cost equation as represented by a straight line.

Now that we have an equation for the total cost, we can proceed to the next problem of expressing the revenue. Let us say that the corporation bills the customer for direct labor plus 120% overhead. We can say that the revenue is expressed by

$$R = L + 1.2L = 2.2L \tag{4}$$

We observe that the revenue is obtained by taking the direct labor and multiplying it by the factor 2.2. The revenue associated with the direct labor L is represented by a second straight line in Figure 2. The difference between the revenue and the cost is the gross profit. This can be written in mathematical form as

$$P = R - C \tag{5}$$

It will be convenient to express the profit for any value of the direct labor, and so we take our Equation (5) and substitute in the expression for the revenue from Equation (4) and for the cost from Equation (3), and get

$$P = 2.2L - (6000 + 1.5L) \tag{6}$$

This can be written as

$$P = .7L - 6000 \tag{7}$$

We see that the profit is obtained by taking .7 times the direct labor and by subtracting $6000, as shown graphically in Figure 3. For instance, if the labor is $20,000, then the profit is $14,000 less $6000, or $8000. The break even point is obtained when the profit is zero, or when

$$L = \frac{6000}{.7} = 8600 \tag{8}$$

We can verify in Figure 3, that the break even point is, indeed, at a direct labor cost of $8600.

Now we can ask the question of what is the formula for the overhead rate? We obtain the overhead rate by taking the expense divided by direct labor, or

$$O_r = \frac{e}{L} \tag{9}$$

We can take our Equation (1) and write

$$O_r = \frac{6000 + \frac{1}{2}L}{L} = \frac{1}{2} + \frac{6000}{L} \tag{10}$$

This is the expression for the overhead rate. A graphical representation of the overhead rate is shown in Figure 4. For instance, when the direct labor is $20,000 we get an overhead rate of 80% which is shown as .8. The diagram clearly shows that, indeed, as direct labor goes up the overhead rate decreases. If we had a very large amount of direct labor, the overhead rate would decrease to 50%.

What is the profit rate for The Alpha Corporation? The profit rate is obtained by taking the profit and dividing it by the revenue, or

$$P_r = \frac{P}{R} = \frac{.7L - 6000}{2.2L} = .318 - \frac{2730}{L} \qquad (11)$$

A graphical representation of the profit rate is shown in Figure 5, and we see that the profit rate is zero at the break even point of $8600, and then it begins to climb to a higher figure. If we had a very large amount of direct labor, then the profit rate would rise up to .318.

The point I am trying to make here is really a very simple one. The mathematical formula expressing the expense as a function of the direct labor, is a simple and convenient way to compute costs and profits. It seems to me that it is easier to work with this equation than with the conventional overhead rate concept when this overhead rate is a variable one. Of course, you may want to compute the overhead rate as this is the traditional way of doing things, however, this is a matter of a simple computation as indicated by Equation (10).

Now, one of the peculiar traits of scientists is that they try to describe phenomenon in a general sort of fashion. Our equation for The Alpha Corporation says that the expense is given by taking one-half of the direct labor, and adding $6000. If we had another corporation with this simple sort of a sales structure, we would expect that these particular numbers in the formula would change. The advantage of mathematics is that this contingency of having these numbers change can be taken care of in a very simple way. Now that all of you are becoming mathematicians, I dare to develop this formula in a general form. I write that indirect labor or expense can be given by the following equation:

$$e = a + bL \tag{12}$$

In this equation, L is again the direct labor, and on the left-hand side e is again the expense. However, you note that instead of writing $6000, I write the letter a, and instead of writing that we need to take one-half of the direct labor, I write the letter b. This is the algebraic notation expressing the fact that the expense is a straight-line function of the direct labor. If, at any time, you feel uncomfortable about these letters, a and b, then you just substitute in the illustrative values of 6000 and .5, and you remember that we are dealing with the same problem as before. However, now we can express the total cost as

$$C = L + e = a + (b + 1)L \tag{13}$$

In our original Equation (3), we had instead of a, the $6000 figure, and instead of the b+1 we had .5 plus 1, which is of course 1.5. Now that we have the cost expressed in algebraic form, let us proceed to express the revenue. We say that r is the overhead rate charged to the customer; revenue is given by

$$R = (1 + r)L \tag{14}$$

(Compare this with Equation (4) where r is 1.2 and r+1 is 2.2.) We can compute the gross profit as the difference between the revenue and the cost and get the equation

$$P = R - C = (r - b)L - a \tag{15a}$$

For The Alpha Corporation, we had a=6000, and b=.5 and, therefore, in that case the profit is given by

$$P = (r - .5)L - 6000 \tag{15b}$$

We recognize here then for different values of r (which correspond to the different billing rates), we get, of course, different values of profits. In Figure 6 we show these profit possibilities by different straight lines, each line corresponds to a different billing rate. This is a planning chart for the management of The Alpha Corporation. Depending on the direct labor forecasted and the billing rate proposed, one can determine what profits could be realized.

Let us obtain now the general expression for the overhead rate. We know that this can be computed by taking the expense and dividing it by the direct labor. Therefore,

$$O_r = \frac{e}{L} = \frac{a + bL}{L} = b + \frac{a}{L} \tag{16}$$

(This is similar to Equation (10) which gives the overhead rate for The Alpha Corporation.) We can compute the profit rate by dividing profit with revenue and so get,

$$P_r = \frac{P}{R} = \frac{(r-b)L - a}{rL} = (1 - \frac{b}{r}) - \frac{a}{rL} \tag{17}$$

For instance, for The Alpha Corporation, b=.5 and a=6000, and so the profit rate is given by

$$P_r = (1 - \frac{.5}{r}) - \frac{6000}{rL} \tag{18}$$

In Figure 7 we show a graphical representation of this equation. When the customer is charged 120% overhead, r=1.2 and we get our profit rate curve shown in Figure 4. Figure 7 can be useful for planning purposes as it shows how the profit rate changes if the billing rate is changed.

When I started to talk about The Alpha Corporation, I said I will develop mathematical formulas to deal with the cost and profit picture. We have done this and in the course of doing so, you have picked up on the way a little bit of mathematics. Now we can proceed to a somewhat more complicated corporation, and so I proceed to describe cost accounting for the hypothetical Beta Corporation.

Cost Accounting in The Beta Corporation

The Beta Corporation is somewhat more complicated than The Alpha Corporation but it is still a highly idealized corporation. There are only two departments, Department 1 and Department 2, and each of these departments has direct and indirect expense. For each of the departments, we can set up an overhead accounting system in the same way as we have done in The Alpha Corporation. However, in The Beta Corporation there is, in addition, general and administrative expense which cannot be directly associated with either Department 1 or Department 2. The graphical representation shown in Figure 8 might help in describing this cost accounting problem. We show direct labor and expense in each of the departments, and then we show in a separate box general and administrative expenses and this is where all expenses are charged which cannot be directly associated with either Department 1 or Department 2.

We denote by L_1 the direct labor in Department 1, and by e_1 the expense in Department 1. By carrying out a statistical analysis similar to the one we have done for The Alpha Corporation, we find that the expense in Department 1 can be computed with the aid of the following formula:

$$e_1 = 1000 + .5L_1 \tag{19}$$

For instance, if the direct labor L_1 is \$15,000 then the expense e_1 is \$8500. The cost of operating Department 1 can be computed by adding the direct labor and the expense, or

$$e_1 = e_1 + L_1 \tag{20}$$

With the aid of Equation (19) this can be written as

$$c_1 = 1000 + 1.5L_1 \tag{21}$$

To illustrate this, if the direct labor L_1 is \$15,000, then the operating cost of Department 1 is \$23,500.

Quite similarly, for Department 2 the expense e_2 can be computed as

$$e_2 = 3000 + .3L_2 \tag{22}$$

where L_2 denotes the direct labor in Department 2. The operating costs of Department 2 can be computed by adding direct labor and expense. Therefore, the operating cost c_2 of Department 2 is given by

$$c_2 = e_2 + L_2 \tag{23}$$

This, again, can be written in the form

$$c_2 = 3000 + 1.3L_2 \tag{24}$$

As an illustration, we can easily compute that if the direct labor in Department 2 is \$5000, then the expense is \$4500 and the departmental expense is (the sum of these two), \$9500.

A graphical representation of the indirect charges in each department is shown in Figure 9. It can be seen, for instance, that when the direct labor is $10,000, then the indirect expense is $6000 in each of the two departments. We recognize however that an increase in direct charge of say $1000 increases expenses in Department 1 by $500, but in Department 2 by $300. The incremental increase in expenses is lower in Department 2 than in Department 1, though the "fixed" expense is lower in Department 1 than in Department 2.

So far, we have established ways of computing expenses in Department 1 and Department 2, but we have not set up the computation for the general and administrative expenses. In Table 2, general and administration expenses for The Beta Corporation are shown for the last sixteen months. The first column shows c_1 the expenses in Department 1, the second column shows c_2 or the expenses in Department 2, and the third column shows E, the general and administrative expenses. The question now is how to relate these general and administrative expenses to the departmental costs c_1 and c_2?

Let us note from Table 2 that in the accounting period 1, 2, 4, and 16, departmental expenses for the first department are $20,000. In the same accounting period, the departmental expenses in Department 2 vary, and might be $6000, $16,000, $10,000 or $14,000. We prepare now a graphical representation as shown in Figure 10. The small crosses show the variation in general and administrative expenses for those accounting periods where the departmental expenses in Department 1 are $20,000. We can approximate the general and administrative expenses with the aid of a straight line as shown in Figure 8, and write

$$E = .30c_2 + 7000 \qquad (25)$$

Let us take now the accounting periods 4, 9, 11, and 14. In these periods the departmental expenses in Department 2 are $10,000. Let us prepare now a diagram as shown in Figure 11. This diagram represents general and administrative expenses for The Beta Corporation for those accounting periods when the departmental costs in Department 2 are $10,000. We can again approximate general and administrative expenses with the aid of a straight line and use the formula

$$E = .20c_1 + 6000 \tag{26}$$

These formulas are useful in computing general and administrative expenses for a few accounting periods. Namely, these formulas work only when departmental costs in Department 1 are $20,000, or when departmental costs in Department 2 are $10,000. The question is now, how can we get a formula that will handle all our accounting periods? The first step in the preparation of such a formula is shown in Figure 12. Here, the horizontal axis is the cost in Department 1, and the vertical axis is the cost in Department 2. Each cross in Figure 12 represents a certain accounting period. Next to each cross we also show a number which gives the general and administrative expenses in that particular accounting period. For instance, we notice that in the fourth accounting period, departmental costs in Department 1 are $20,000, departmental costs in Department 2 are $10,000, and general and administrative expenses are $10,000 as shown by the number written next to this cross. A better geometrical representation of this situation is shown in Figure 13 where each accounting period is represented by a dot, but the general and administrative expense is shown by a vertical rod which has the length of the general and administrative expense. We have now a dot in three dimensional space corresponding to each accounting period, and our problem is to summarize these data by a mathematical formula. A convenient way to do this is shown in Figure 14, where the plane is adjusted so that it goes close to each of the dots in our three dimensional space. We say that this plane approximates general and administrative expenses for The Beta Corporation. Those of you who have a little knowledge of analytic geometry will not be surprised when I say that points on this plane can be computed with the aid of the following equation:

$$E = .20c_1 + .30c_2 + 3000 \tag{27}$$

This is, then, the equation that we use to compute general and administrative expenses for The Beta Corporation. By a simple computation you can convince yourself that this equation includes as special cases the previous equations where we computed general and administrative expenses under the condition that departmental costs in Department 1 were $20,000, or when departmental costs in Department 2 were $10,000. In addition, this formula allows us to compute general and administrative expenses for any other accounting period. For instance, if departmental costs are $16,000 in Department 1 and $6000 in Department 2, then we can compute that the general and administrative expenses are $8000.

Let us notice here that if departmental expenses in Department 1 go up by $1000, then general and administrative expenses will go up by $200. However, if departmental costs in Department 2 go up by $1000, then the general and administrative expenses go up by $300. This means then that an increment of expenses in Department 2 involve a greater increase in general and administrative expenses than an incremental increase in Department 1.

Suppose the direct labor cost is $15,000 in Department 1, and $5000 in Department 2, what would the general and administrative expense be? We have already computed that costs in Department 1 would be \$23,500, and costs in Department 2 would be \$9500. Therefore, from Equation (27) we get that the general and administrative expenses are \$10,550.

Our graphical representation shown in Figure 14 is somewhat inconvenient because it is a three dimensional representation. An alternate way of representing the general and administrative expenses formula is shown in Figure 15. The horizontal axis shows costs in Department 1, the vertical axis shows costs in Department 2. Each of the straight lines represent a fixed general and administrative expense line for The Beta Corporation. For instance, we can see from the diagram that if costs in Department 1 are \$16,000 and costs in Department 2 are \$6000, then the general and administrative expense is \$8000. This is, then, a more convenient graphical way of summarizing general and administrative expenses for The Beta Corporation.

Let us compute now the total corporate costs. This is given by adding the costs in Department 1, in Department 2, and finally, the general and administrative expenses. If we denote by C the corporate costs for The Beta Corporation, we have

$$C = c_1 + c_2 + E \tag{28}$$

We can use our expression for the general and administrative costs, Equation (27), and get

$$C = 1.2c_1 + 1.3c_2 + 3000 \tag{29}$$

This is, then, the formula for corporate costs in The Beta Corporation. For instance, in the case mentioned before, costs in Department 1 were \$23,500, and in Department 2 were \$9500. From Equation (29) we get that corporate costs for The Beta Corporation are \$43,550.

We can also express the corporate cost with the aid of the direct labor cost, by using our Equations (21) and (24). We get

$$C = 1.2(1000 + 1.5L_1) + 1.3(3000 + 1.3L_2) + 3000 \tag{30}$$

which can be written in the form

$$C = 1.8L_1 + 1.69L_2 + 8100 \tag{31}$$

For instance, if direct labor in Department 1 is \$15,000, and in Department 2 is \$5000, the total cost for the corporation is \$43,550. (This, of course, agrees with our previous computation.)

We notice from our equation for the corporate costs, that a \$1000 increase in labor in Department 1 increases corporate costs by \$1800. A \$1000 increase in direct labor in Department 2 increases corporate costs by \$1690. We could say then that incremental corporate overhead is 80% for Department 1, and 69% for Department 2. On the other hand, it follows from Equations (19) and (22), that the departmental (incremental) overhead is 50% in Department 1, and 30% in Department 2. Furthermore, we can see from Equation (27) that (incremental) general and administrative expenses amount to 20% for departmental costs in Department 1, and 30% in Department 2. Let us, however, remind ourselves that these incremental costs represent only the variable part of overhead. As far as total costs are concerned, we must use our equations in full to take into account the "fixed" part of the overhead costs.

Let us look now at the profit picture of The Beta Corporation. First, let us compute the revenue. Suppose Department 1 has such contracts that for each

\$1000 of direct labor, \$3000 is billed. On the other hand, Department 2 collects only \$2500 for each \$1000 of direct labor. In a mathematical form, the revenue for The Beta Corporation is given by

$$R = 3L_1 + 2.5L_2 \tag{32}$$

For instance, if direct labor is \$15,000 in Department 1, and \$5000 in Department 2, then the corporate revenue is \$57,500. The gross profit is obtained by taking the revenue less the corporate cost. Mathematically this can be expressed as

$$P = R - C \tag{33}$$

With the aid of our expressions for corporate cost and corporate revenue, this can be written as

$$P = 3L_1 + 2.5L_2 - (1.8L_1 + 1.69L_2 + 8100) \tag{34a}$$

or as,

$$P = 1.2L_1 + .81L_2 - 8100 \tag{34b}$$

A graphical representation of this profit is shown in Figure 16. The horizontal axis is direct labor in Department 1, the vertical axis is direct labor in Department 2. Along each of the straight lines the profit is fixed. For instance, if direct labor is \$16,000 in Department 1, and \$5000 in Department 2, then the profit is \$15,000. We realize that an increase of \$1000 of direct labor in Department 1 results in an increase of profit of \$1200, whereas the same increase in direct labor in Department 2 leads to an increase of profit of \$810. For instance, if management has to make a decision of where to put in an additional \$1000 of labor, then this increase should be introduced in Department 1 as this will result in a higher profit.

So far, we have used either graphical or mathematical representation in describing the accounting situation in The Beta Corporation. Now, we want to discuss another method of presentation which can be more convenient, particularly when the cost accounting problem gets more complex.

In Figure 17 we show a description of the accounting situation for The Beta Corporation. Each accounting entity shown in the first column can be computed with aid of the accounting information listed in the top row. For instance, the departmental expense in Department 1, e_1, is computed by taking \$1000 and adding .5 times the direct labor in Department 1, which is listed under the column L_1. The departmental cost in Department 1 can be computed by taking 1 times the direct labor cost in Department 1, L_1, and adding it to 1 times the departmental expense e_1. The way we compute any accounting entity, as shown in the first column, is to take the numbers in the same row and multiply them with the proper heading shown in the top row. For instance, general and administrative expenses, E, can be computed by taking 3000 times 1 plus .2 times c_1 plus .3 times c_2. Here, we have a tabular representation that has the advantage that it summarizes all of our equations and computational methods. We will see later that this sort of a representation is particularly useful in more complicated cost accounting problems.

Before we leave our hypothetical Beta Corporation, we develop the cost accounting system for a corporation of this type under more general conditions. We assume again that this type of corporation has two departments and also it has general and administrative expenses, but we do not assume the particular rate of overhead in each of these departments or for the corporation. We say that expenses in Department 1 can be computed from the equation

$$e_1 = a_1 + b_1L_1 \tag{35}$$

and expenses in the second department can be computed from

$$e_2 = a_2 + b_2L_2 \tag{36}$$

Departmental costs for Department 1 can be computed from

$$c_1 = e_1 + L_1 = a_1 + (1 + b_1)L_1 \tag{37}$$

and departmental costs for Department 2 can be computed from

$$c_2 = e_2 + L_2 = a_2 + (1 + b_2)L_2 \tag{38}$$

General and administrative expenses are related to departmental costs with the aid of the formula

$$E = A_1c_1 + A_2c_2 + B \tag{39}$$

Total corporate costs are given by

$$C = c_1 + c_2 + E \tag{40}$$

This can also be written as

$$C = \left[(1 + b_1)(1 + A_1)L_1 + (1 + b_2)(1 + A_2)L_2\right] + \\ + \left[B + a_1(1 + A_1) + a_2(1 + A_2)\right] \tag{41}$$

We recognize here that the first term is the variable cost related to direct labor charges in the departments, and the second is a fixed cost term. We assume again that the customer is billed at a fixed rate for each direct labor charge. We say that this chargeable overhead rate is r_1 for Department 1, and r_2 for Department 2. This means that total corporate revenue is given by

$$R = (1 + r_1)L_1 + (1 + r_2)L_2 \tag{42}$$

In case you have any difficulty in following these more abstract equations, I suggest that you compare them with the previous equations and determine what the value of A_1, B_1, A_2, B_2, and so on, is for The Beta Corporation. The profit is computed by taking the revenue less the cost, or

$$P = R - C \tag{43}$$

This can be written now as

$$P = \left[(1 + r_1) - (1 + b_1)(1 + A_1)\right] L_1 + \left[(1 + r_2) - (1 + b_2)(1 + A_2)\right] L_2 \tag{44}$$
$$- \left[B + a_1(1 + a_1) + a_2(1 + A_2)\right]$$

We recognize that the first two terms represent the variable part of the profit, whereas the last term represents a fixed term. This last equation is a generalization of the profit equation shown by Equation (34b).

These equations are somewhat complicated and not too easy to remember. A good way to summarize all our equations is shown again by Figure 18. The rules of computing the accounting entities shown in the first column are again the same as in Figure 17. For instance, corporate costs, C, can be computed by taking 1 times departmental cost, c_1, plus 1 times departmental cost, c_2, plus 1 times the corporate expense, E. This is a convenient summarization of the cost accounting system in a corporation which is of the same type as The Beta Corporation.

Cost Accounting in The Gamma Corporation

Now we know enough mathematics to consider a more complicated corporate structure. As shown in Figure 19, The Gamma Corporation has three divisions, and each of these divisions has several departments. There is direct labor and expense in each of these departments just the same as in The Alpha or Beta Corporations. In addition there are administrative expenses in each of the three divisions. Finally, we have general and administrative expenses on the corporate level. In order to develop a cost accounting system, we follow the method we used in The Alpha or Beta Corporations. Let us say that in Department i, direct labor costs are L_i. This is a new trick in our mathematical notation. The small letter i denotes that our notation refers to Department 1, or Department 2, all the way up to Department 9, or as the mathematician would say, i can take the value of 1 to 9. Then we can say that departmental expenses are to be computed with the aid of a straight line formula, or

$$e_i = a_i + b_i L_i \qquad (45)$$

If you are still perplexed by the subscript i, then instead of i put, say, the number 2 and then you get the departmental expenses for Department 2. Departmental costs are obtained by adding direct labor to expenses, and so we get

$$c_i = e_i + L_i = a_i + (1 + b_i)L_i \qquad (46)$$

We have now the departmental costs for each of the departments and we can proceed to determine divisional expense, say, for the first division. You recall that in The Beta Corporation we did this sort of a computation but in that case we had only two departments, whereas here we have three departments. Consequently, our equation for the divisional expense is somewhat more complicated and can be written in the following form:

$$E_1 = A_1c_1 + A_2c_2 + A_3c_3 + B_1 \tag{47}$$

Total divisional costs are obtained by adding the departmental costs to the divisional expense. For instance, the divisional cost for Division 1 is given by the equation

$$C_1 = c_1 + c_2 + c_3 + E_1 \tag{48}$$

Quite similarly, we can develop divisional costs for Division 2 and Division 3.

The next problem now is to develop an equation for the general and administrative expenses for The Gamma Corporation. These will be related to the divisional costs and following our previous examples, we say that the general and administrative expenses for The Gamma Corporation can be written in the form

$$E_c = f_o + f_1C_1 + f_2C_2 + f_3C_3 \tag{49}$$

where f_o, f_1, f_2, and f_3 are similar constants as the ones appearing in Equation (27). Corporate costs are obtained by adding divisional costs and general and administrative costs, or

$$C_c = C_1 + C_2 + C_3 + E_c \tag{50}$$

The corporate revenue can be computed by adding the revenues for each of the departments. We denote the revenue for Department 1 by R_1, for Department 2 by R_2, and so on, and then we write that the corporate revenue is given by

$$R_c = R_1 + R_2 + R_3 + \cdots + R_9 \tag{51}$$

We again assume that each department bills its customers at a fixed rate of the direct labor and, therefore, we say that the corporate revenue is given by

$$R_c = (1 + r_1)L_1 + (1 + r_2)L_2 + (1 + r_3)L_3 + \cdots + (1 + r_9)L_9 \tag{52}$$

On the right-hand side we have 9 terms corresponding to the revenues in each of the departments. Mathematicians have an abbreviated way of writing equations of this type. Namely, they write

$$R_c = \sum_{i=1}^{9} (1 + r_i)L_i \tag{53}$$

The greek letter sigma is an abbreviation for the word "summation" and the little letter below the capital sigma says that the summation is to be started at the first department and the letter 9 on the top shows that the summation is to be ended by the ninth department. The profit for the corporation is obtained by taking the corporate revenue less the corporate cost, or

$$P_c = R_c - C_c \tag{54}$$

With the aid of these equations we can compute for any direct labor distribution the departmental, divisional, and corporate expenses. Also, we can compute corporate revenue and corporate profit. We can say then that we have established a system of equations that describe cost accounting in The Gamma Corporation. However, it may be useful to express all these costs, revenues, and profits with the aid of direct labor as these labor costs are the important control variables for The Gamma Corporation. In mathematical language, we say that it will be useful to develop these equations using direct labor costs as independent variables. To do this requires a certain amount of algebra and those of you who are not interested in these mathematical details can skip this part of the presentation and go directly to the next section.

The corporate expenses can be computed from Equation (49) as

$$E_c = f_o + f_1(c_1 + c_2 + c_3 + E_1) + f_2(c_4 + c_5 + E_2) + \\ + f_3(c_6 + c_7 + c_8 + c_9 + E_3) \qquad (55)$$

This can also be written as

$$E_c = f_o + f_1(c_1 + c_2 + c_3 + A_1c_1 + A_2c_2 + A_3c_3 + B_1) + \\ + f_2(c_4 + c_5 + A_4c_4 + A_5c_5 + B_2) + \qquad (56) \\ + f_3(c_6 + c_7 + c_8 + c_9 + A_6c_6 + A_7c_7 + A_8c_8 + A_9c_9 + B_3)$$

We introduce now the notation

$$g_1 = g_2 = g_3 = f_1 \tag{57}$$

$$g_4 = g_5 = f_2 \tag{58}$$

$$g_6 = g_7 = g_8 = g_9 = f_3 \tag{59}$$

With the aid of these equations, corporate expenses can be expressed as

$$E_c = f_o + \sum_{i=1}^{3} f_i B_i + \sum_{i=1}^{9} g_i (1 + A_i) c_i \tag{60}$$

This can also be written as

$$C_c = \sum_{i=1}^{9} c_i + \sum_{i=1}^{3} E_i + E_c \tag{61}$$

or

$$C_c = \sum_{i=1}^{9} c_i + \sum_{i=1}^{9} A_i c_i + \sum_{i=1}^{3} B_i + E_c \tag{62}$$

or

$$c_c = f_o + \sum_{i=1}^{3} (1 + f_i) B_i + \sum_{i=1}^{9} (1 + g_i)(1 + A_i) c_i \tag{63}$$

Finally, the corporate expense can be expressed as

$$C_c = \left[f_o + \sum_{i=1}^{3} (1 + f_i) B_i + \sum_{i=1}^{9} (1 + g_i)(1 + A_i) a_i \right] + \left[\sum_{i=1}^{9} (1 + g_i)(1 + A_i)(1 + b_i) L_i \right] \tag{64}$$

We recognize in this last equation that the first term is a fixed corporate expense, whereas the second term represents the corporate expense which is proportional to direct labor. With the aid of this equation, the profit for The Gamma Corporation can be expressed as

$$P_c = \left[\sum_{i=1}^{9} \left[(1 + r_i) - (1 + g_i)(1 + A_i)(1 + b_i) \right] L_i \right] - \left[f_o + \sum_{i=1}^{3} (1 + f_i) B_i + \sum_{i=1}^{9} (1 + g_i)(1 + A_i) a_i \right] \quad (65)$$

We recognize again that the first term is the profit proportional to the direct labor, whereas the second term (which is a negative one) represents a fixed amount. If we consider only the variable term, (that is, the first term), then we recognize that this proportional part of the profit may be computed as the difference between two terms. The positive term represents the revenue, as r_i is the chargeable overhead rate. The negative term represents the proportional part of the cost. We recognize that there are three levels of overhead for each dollar of direct labor. The highest level of overhead is represented by g_i, the proportional part of the general and administrative expense of The Gamma Corporation. The middle level of overhead is represented by A_i and this is the overhead rate applied to each department to cover divisional expenses. Finally, the lowest level of overhead is represented by b_i which is the expense incurred in each department where the direct labor is expended. We can see then that these various overhead rates cascade into the over-all overhead rate of The Gamma Corporation.

Those of you who wish to better understand this analysis may find it useful to experiment with this formula by using different percentage values for the various overhead rates. This will give you a better feeling for the possible use of these equations.

We have developed now the mathematical equation describing cost accounting in The Gamma Corporation. A convenient way of summarizing these equations is to use the cost accounting table of The Gamma Corporation as shown in Figure 20. This cost accounting table is very similar to the one shown in Figure 18 which relates to corporations similar to The Beta Corporation. Here we broke the table into two different tables, the first one is the divisional cost accounting system, and the second one, as shown on the bottom, is the corporate cost accounting system. Just the same way as before, we can compute any of the accounting entities shown in the first column with the aid of the quantities shown in the top row. For instance, e_3, that is, expenses in Department 3, can be computed by taking a_3 times 1 and adding b_3 times L_3.

An Allocation Problem

In all these cost accounting problems, we assumed that the revenue in each of these departments or divisions is proportional to the direct labor expended. This was due to the fact that we assumed that the customer is charged at a fixed overhead rate. As far as costs are concerned, we worked with departmental, divisional, or corporate costs and we compute all the expenses for the corporation. However, we made no attempt to allocate these expenses to each of the departments, and we made no effort to allocate profits to each of the departments. Let us now return to The Beta Corporation where we have formulas to compute revenues for each of the two departments.

and formulas to compute expenses for the corporation. What are the profits to be allocated to each of these departments?

Let us say that direct labor in Department 1 is $15,000 and direct labor in Department 2 is $5000. We can easily compute from the equations that profits for The Beta Corporation are $13,950. Can we determine which of the departments is more profitable? Let us say that we have $5000 of additional labor available. If we put the $5000 labor into Department 1, then the corporate profit goes up by $6000, whereas if we put the $5000 into Department 2, corporate profits go up only by $4040. Therefore, we can see that by putting the extra $5000 labor into Department 1, we do $960 better. So far, so good. However, our executives may want to know how much Department 1 contributes to profits and how much Department 2 contributes. Let us try to answer this question by allocating costs and revenues to each of these two departments.

We have our Equation (27) which expresses corporate expenses. A part of these expenses are proportional to costs in Department 1 and in Department 2, therefore, it is natural to allocate these expenses to the respective departments. However, what to do with the fixed expense of $3000? Let us say that we allocate these $3000 in proportion to departmental costs. We need to divide the $3000 in two parts and these two parts must be proportional to the departmental costs c_1 and c_2. We can put this into a mathematical formula by saying that the allocated expense to Department 1 is given by

$$E_1 = .20c_1 + \frac{3000}{c_1 + c_2} c_1 \tag{66}$$

The first term is the general and administrative expense proportional to departmental costs in Department 1, and the second term is the part of the $3000 that is allocated to Department 1. Similarly, we compute the expenses for Department 2 as

$$E_2 = .30c_2 + \frac{3000}{c_1 + c_2} c_2 \tag{67}$$

Now, we have formulas to compute departmental expenses. Let us carry out these computations for a hypothetical $15,000 of direct labor in Department 1 and $5000 direct labor in Department 2. In Table 3, in the first row, we show direct labor in each of these departments. The second row shows departmental expenses which include direct labor and departmental overhead, and the third row shows the sum of these two costs. The fourth row shows general and administrative expenses which can be directly allocated to these departments. The fifth row shows that out of the $3000 we allocate $2140 to Department 1, and $860 to Department 2. The sixth row shows general and administrative expenses for each department. (These are obtained by adding the two previous rows.) By adding the third and sixth rows we get total departmental cost in row seven; $30,340 for Department 1 and $13,210 for Department 2. The eighth row shows revenues for each department; $45,000 for Department 1 and $12,500 for Department 2. Finally, the last row in Table 3 shows the profit in each of these departments; a profit of $10,660 for Department 1 and a loss of $710 for Department 2. The combined profit for the corporation is $14,660 less $710 which is $13,950. This is, of course, in agreement with our previous calculations of corporate profits which we obtained without allocating profits to each department.

Now prepare yourself for a great deal of excitement. The manager of Department 2 will strongly object to this loss of $710. He will say that the allocation of the $860 of the $3000 "fixed" general and administrative expenses is most "unfair." In fact, the manager of Department 2 will feel that he should not be burdened at all with this $3000, as he can dispense with general and administrative overhead. He feels that his department is not a losing one. Maybe his department is not very profitable, but it still does break even.

So far, we have only one dissatisfied customer for our cost allocation system. Suppose now, however, that another $15,000 of direct labor is available to The Beta Corporation, and this direct labor is put into enlarging Department 1, as this department brings in more profit. From our profit equation we can easily compute that, under this new condition, (we assume that there is still $5000 direct labor in Department 2), the corporate profits will be $31,950. This means an increase in profits of $18,000 for The Beta Corporation. However, our executives may not be satisfied with this statement, as they want to compute the profit for each of the two departments. In Table 4 we show details of these calculations. We notice that departmental costs in Department 1 increase to $46,000, whereas departmental costs in Department 2 stay at the level of $9500. The fifth row in Table 4 shows how the $3000 (the fixed part of the general and administrative expenses) is allocated in this case. We find that Department 1 is burdened by $2500, and Department 2 by $500. (This distributes the $3000 in the ratio of the direct departmental costs of $46,000 and $9500.) In the seventh row of Table 4 we show total departmental costs as $57,700 and $12,850. The last row shows that profits for Department 1 are $32,300 and that Department 2 has a deficit of $350. We see then that the $15,000 of direct labor in Department 1 has increased corporate profits by $18,000. However, only $17,640 of this is allocated to Department 1 and the rest of the $360 is allocated to Department 2. (This decreases the deficit in Department 2.) At this point, the manager of Department 1 gets really disturbed. He says that Department 2 has not done anything different from before, and still his deficit has decreased. The manager of Department 1 feels that corporate profits went up by $18,000 and this is due solely to his effort of expanding Department 1. We have constructed a synthetic example of cost accounting charged with emotions.

Of course, you can readily see that this sort of an accounting system in a real life situation can get very complicated and disturbing for managers and executives. Now you may ask me the pertinent question, how is it possible that with the aid of all these scientific and mathematical concepts, I have reached such a frustrating dead-end.

I should apologize to you at this point because I have deliberately set a trap and misled you with a pseudo-scientific argument. Until I talked about the problem of allocating costs and profits, everything was built on solid ground. I used past data on the various overhead figures, straight-line approximations to summarize these past records, and developed mathematical equations to help in computing these various overheads. However, when I started to talk about allocating costs and profits I used a very arbitrary concept. I said that the "fixed" part of the general and administrative expense (which is $3000 for The Beta Corporation) is allocated to each of the departments in proportion to departmental costs. However, did I have any reason to do so? If you recall the argument I used there, you will realize that I have used no argument at all. This is, of course, arbitrary and irrational. Now as soon as you begin to do things that are not well justifiable, the results will not be justifiable either. The results of our computation are as good as our assumptions, and if we assume a particular way of allocating the fixed part of costs, then we may not be better off than if we allocate profits arbitrarily. If even one step in our calculation is not justifiable, then the end result is not justified either. I have written equations to express allocable costs, but these equations are unjustified because the assumption in these equations are unjustified. You cannot possibly get a result out of a mathematical argument unless you base it on a sensible foundation. My discussion of this allocation problem is not an illustration of the use of mathematics in accounting, but an illustration of the misuse of mathematics.

After this somewhat elaborate warning that you must not believe mathematical results just because they look abstruse, you may ask me the question of what should we do here? First of all, let us recognize that we do not have data here to determine how general and administrative costs should be allocated. If we have no data then it makes no sense to try to allocate these expenses. What we need is to make a further study and to find some ways to get the facts which tell us how to allocate general and administrative expenses. Once an understanding of the problem is obtained, the proper mathematical formulation of the allocation problem can be developed and a satisfactory solution obtained. We can say here, that the mathematical model presented does not include a method of allocating costs and profits and, therefore, the mathematical model must be enlarged to include a solution of the allocation problem.

Now that I have shown you some possible uses and misuses of mathematical methods in cost accounting, I would like to summarize in a few words the message I have been trying to develop.

Concluding Remarks

First, of all I have tried to explain that I do not think that it is necessary to define operations research or scientific methodology. The important point is that scientists are beginning to get into industry and business and they are making contributions in these fields. Occasionally, scientists use strange methods, though I expect that the bulk of our work in business will not be different from the type you are accustomed to, as after all, even scientists are human. Some of our strange methods will look weird to you, and in particular, the use of mathematics can be perplexing. I tried to show through some examples from the physical sciences that there is good reason to use mathematics there. As far as cost accounting is concerned, I constructed some hypothetical corporations, and have shown how overhead computations could be made with the aid of mathematical equations. I hasten to add that these cost accounting equations I use only as illustrations, and I do not seriously propose that you go ahead and use these equations without modifications. I believe that a great deal more work will have to be done before these mathematical equations can be applied. What I really tried to accomplish was to give you an insight into the methods that scientists might use in the field of accounting.

I am concluding this speech in the hope that when you encounter scientists in your business, you will encourage them and show a sympathetic understanding of their efforts towards solving some of the knotty problems of cost accounting.

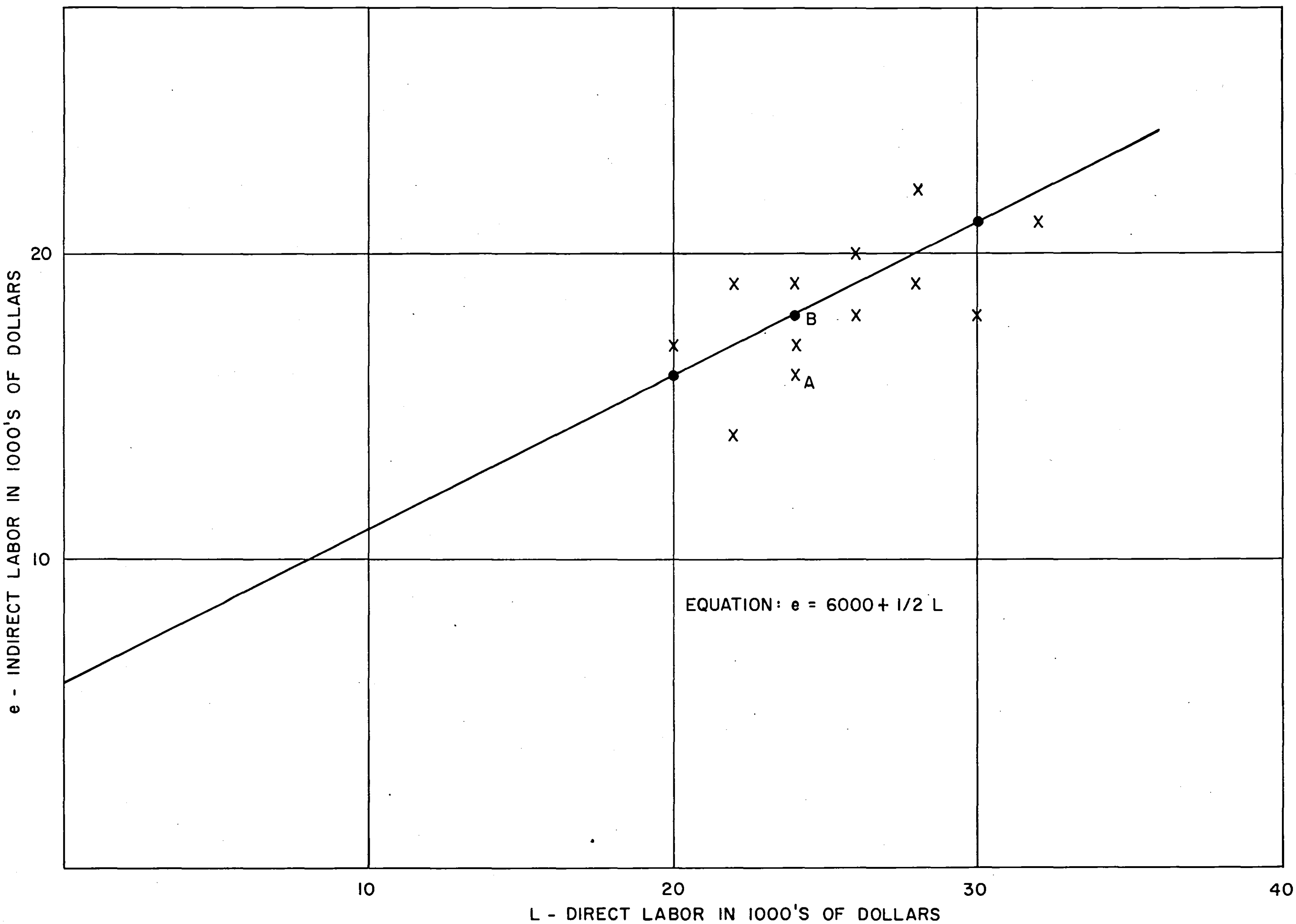


FIG. I. INDIRECT EXPENSES IN THE ALPHA CORPORATION

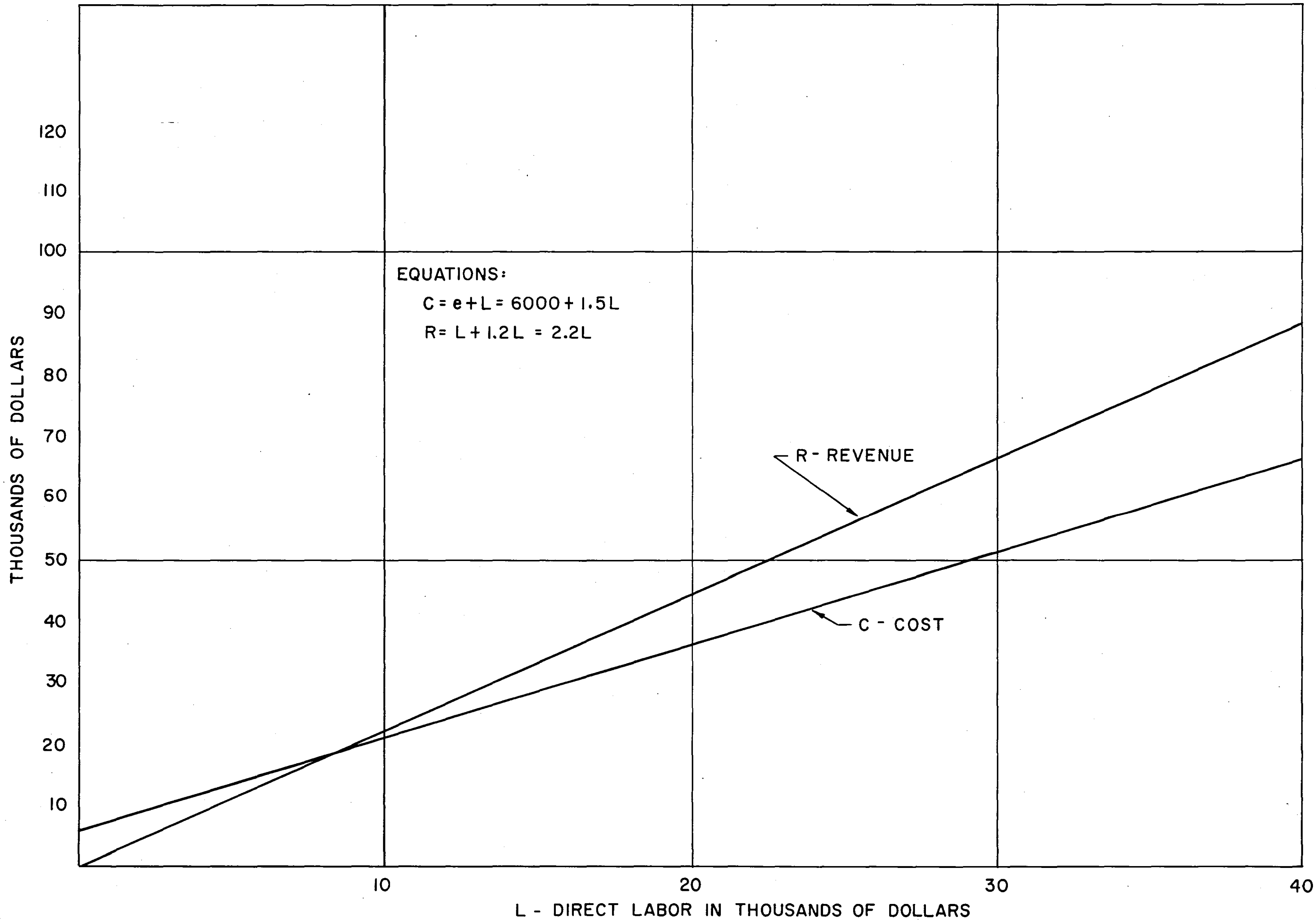


FIG. 2. COST AND REVENUE FOR THE ALPHA CORPORATION

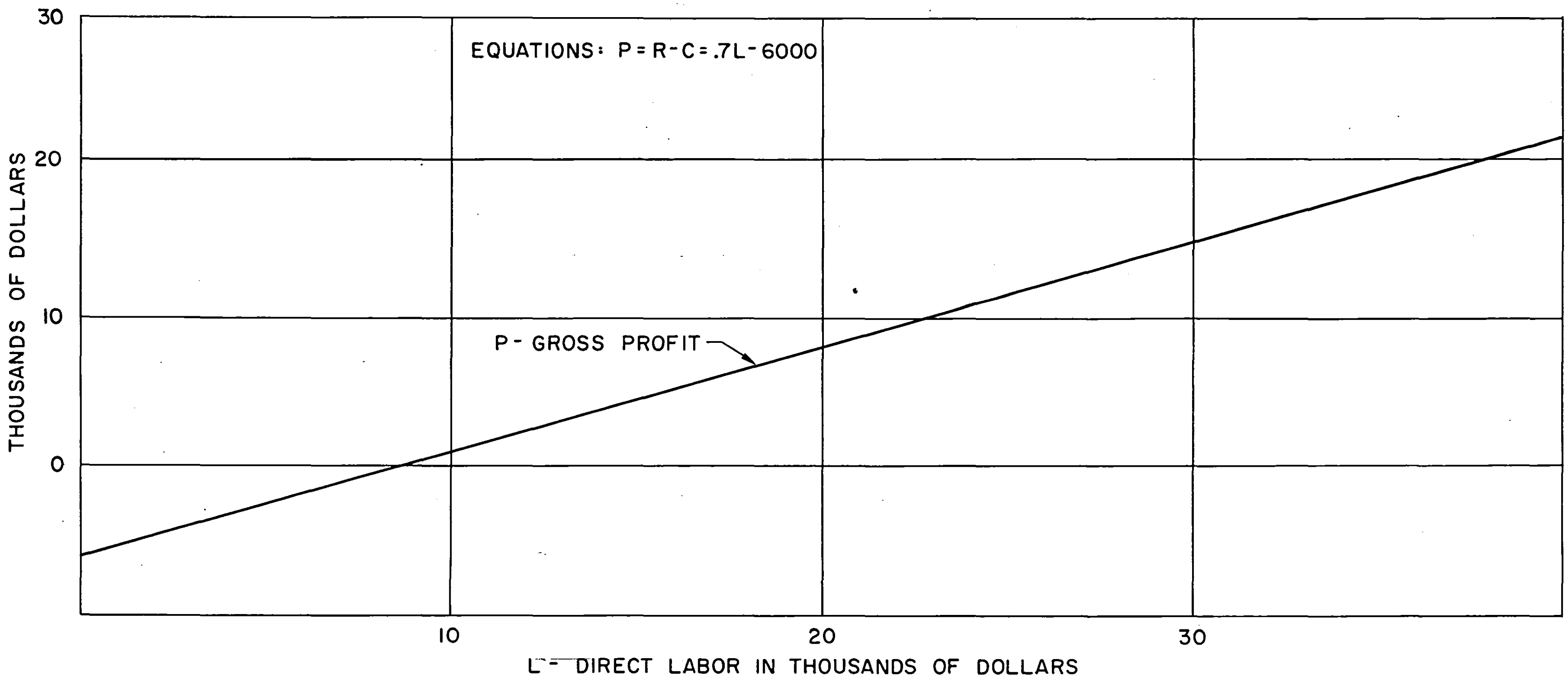


FIG. 3. PROFIT FOR THE ALPHA CORPORATION

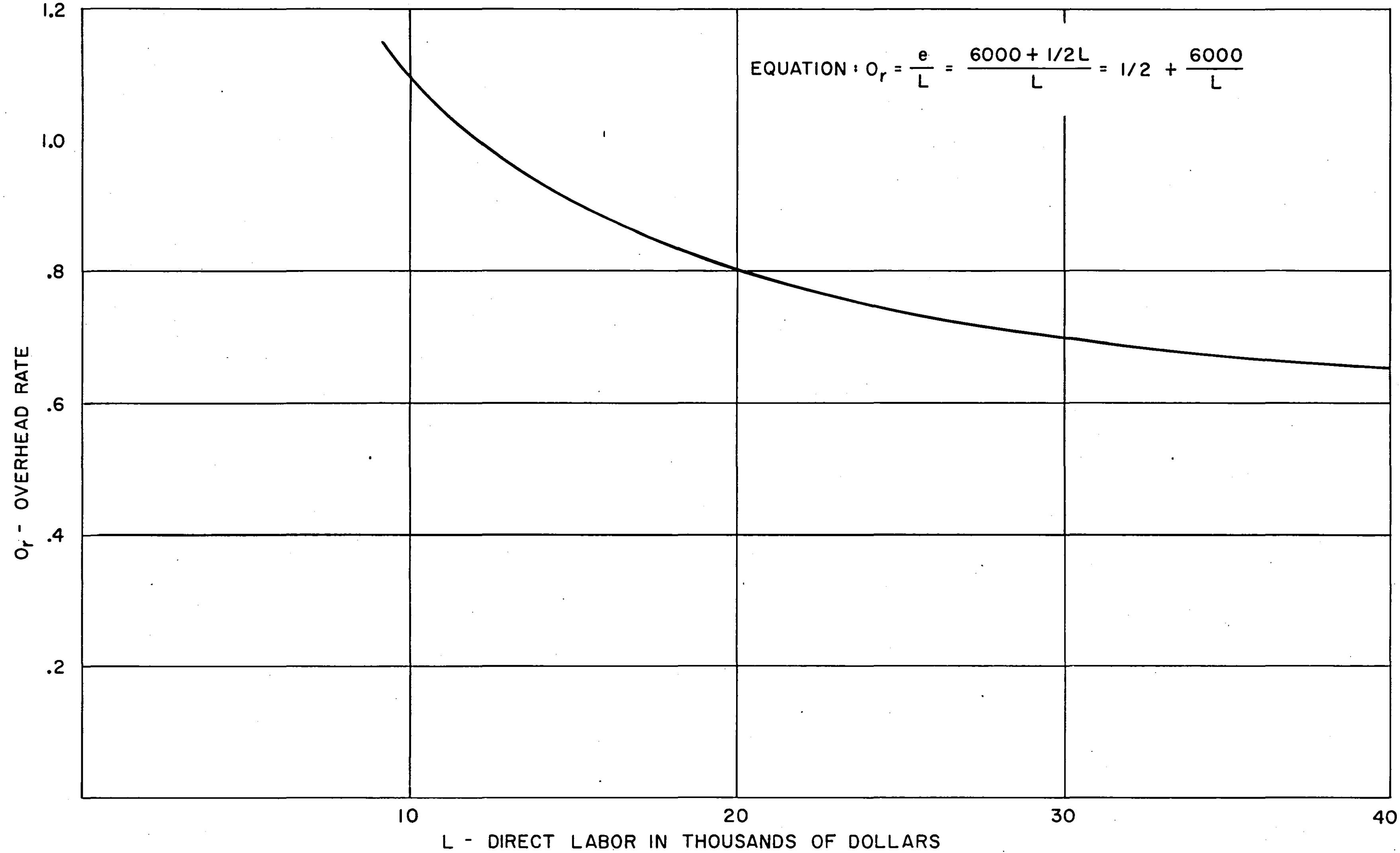


FIG. 4. OVERHEAD RATE FOR THE ALPHA CORPORATION

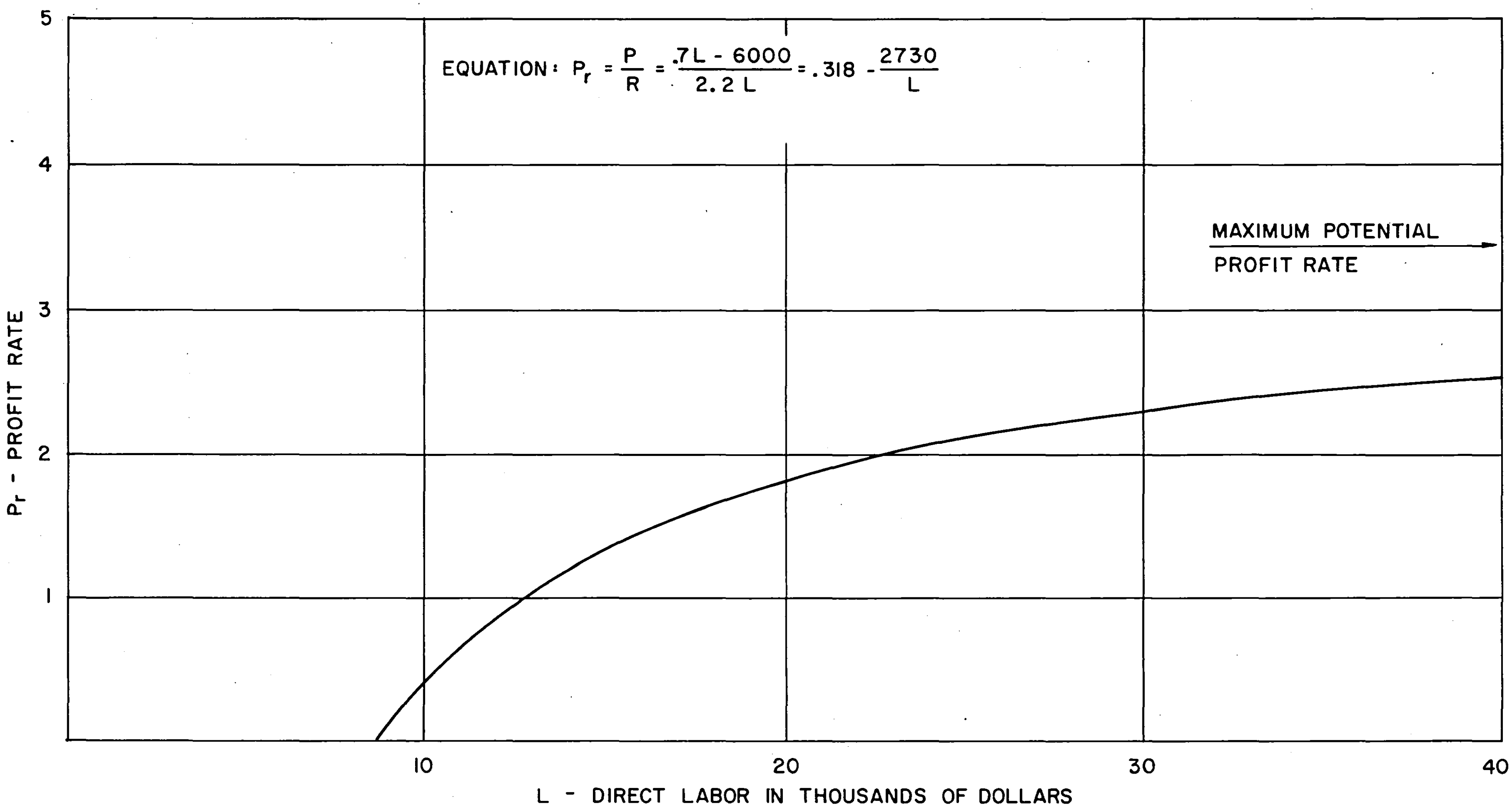


FIG. 5. PROFIT RATE FOR THE ALPHA CORPORATION

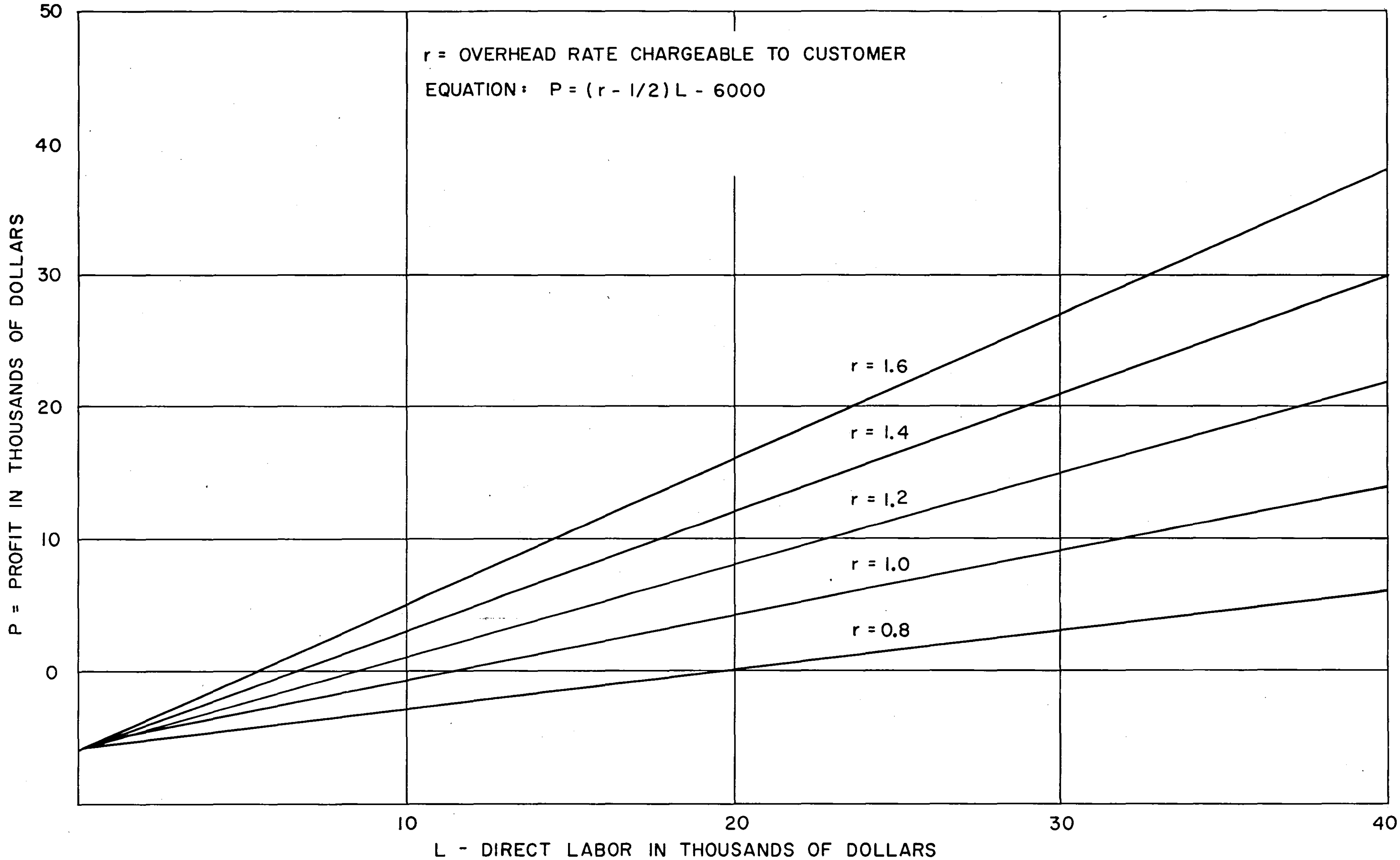


FIG. 6. PROFIT POSSIBILITIES FOR THE ALPHA CORPORATION

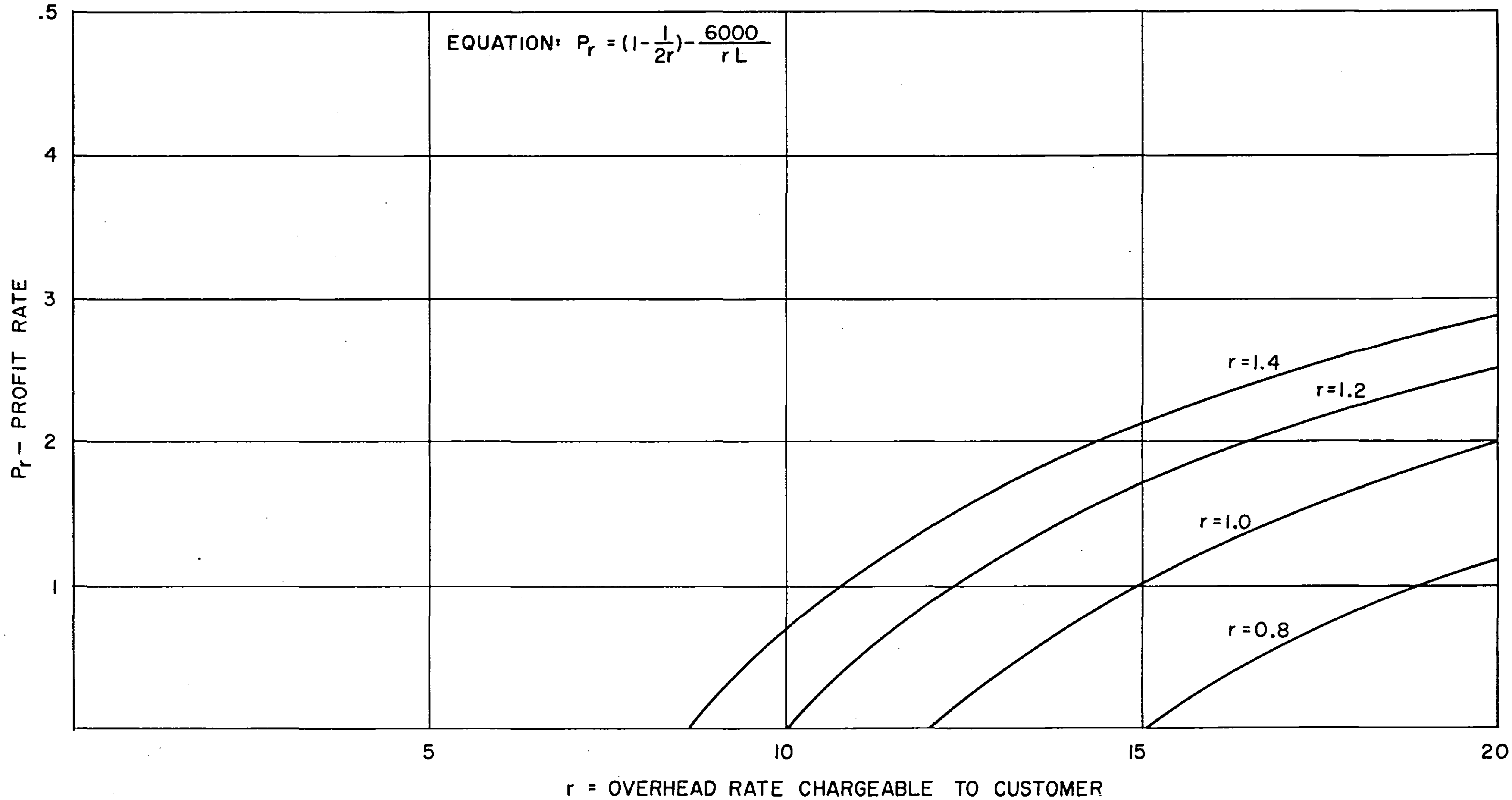


FIG. 7. PROFIT RATE POSSIBILITIES FOR THE ALPHA CORPORATION

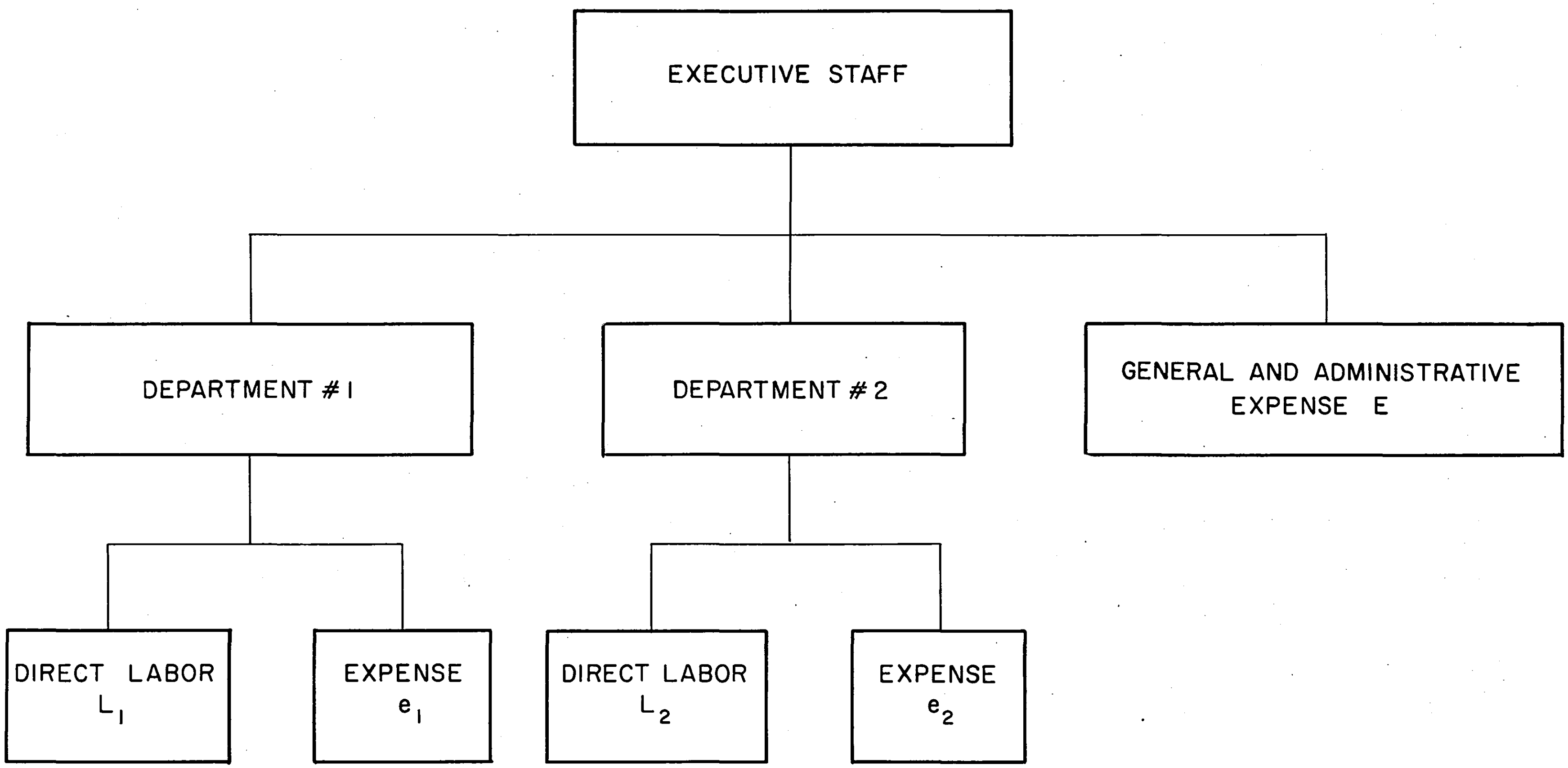


FIG. 8 ORGANIZATION OF THE BETA CORPORATION

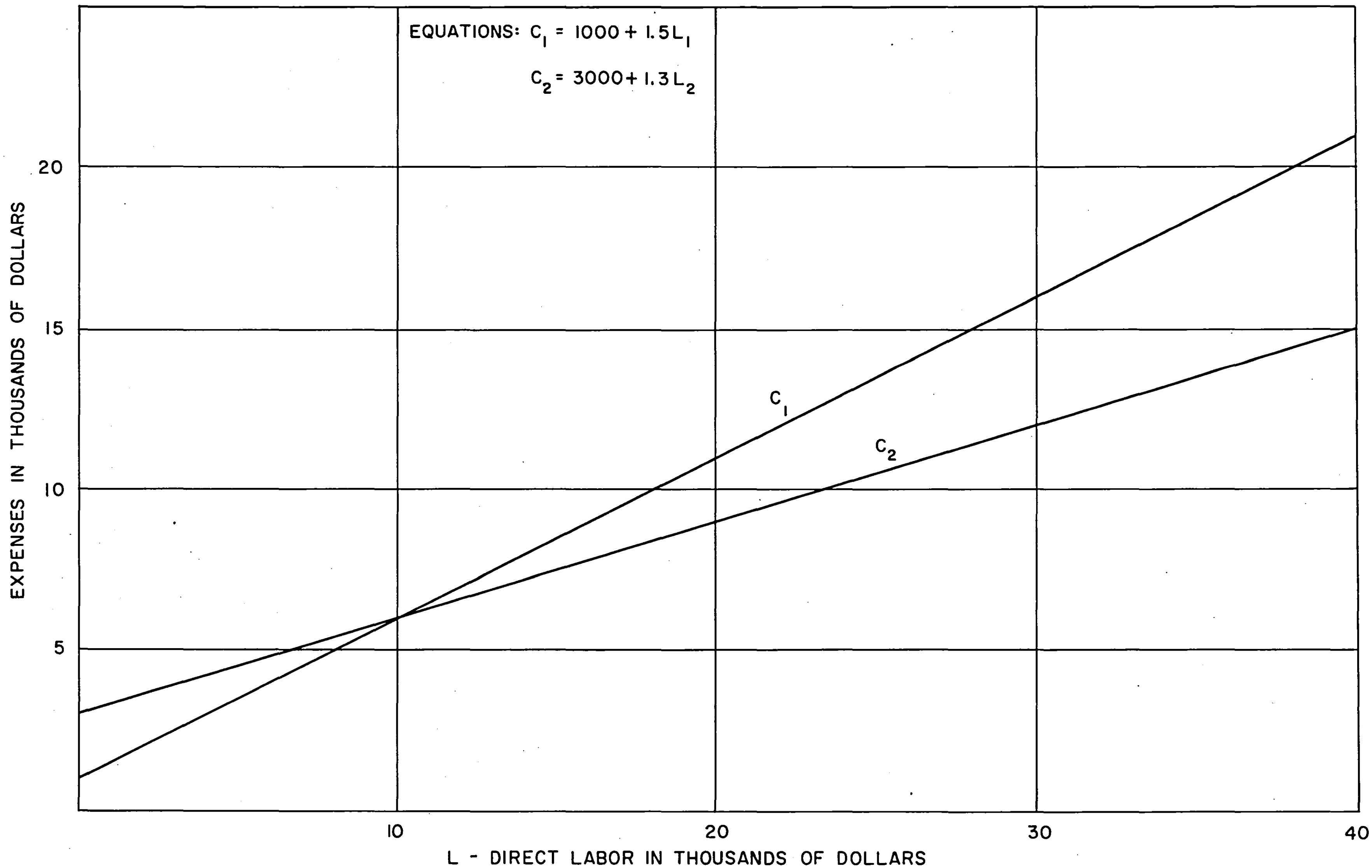


FIG. 9. DEPARTMENTAL EXPENSES IN THE BETA CORPORATION

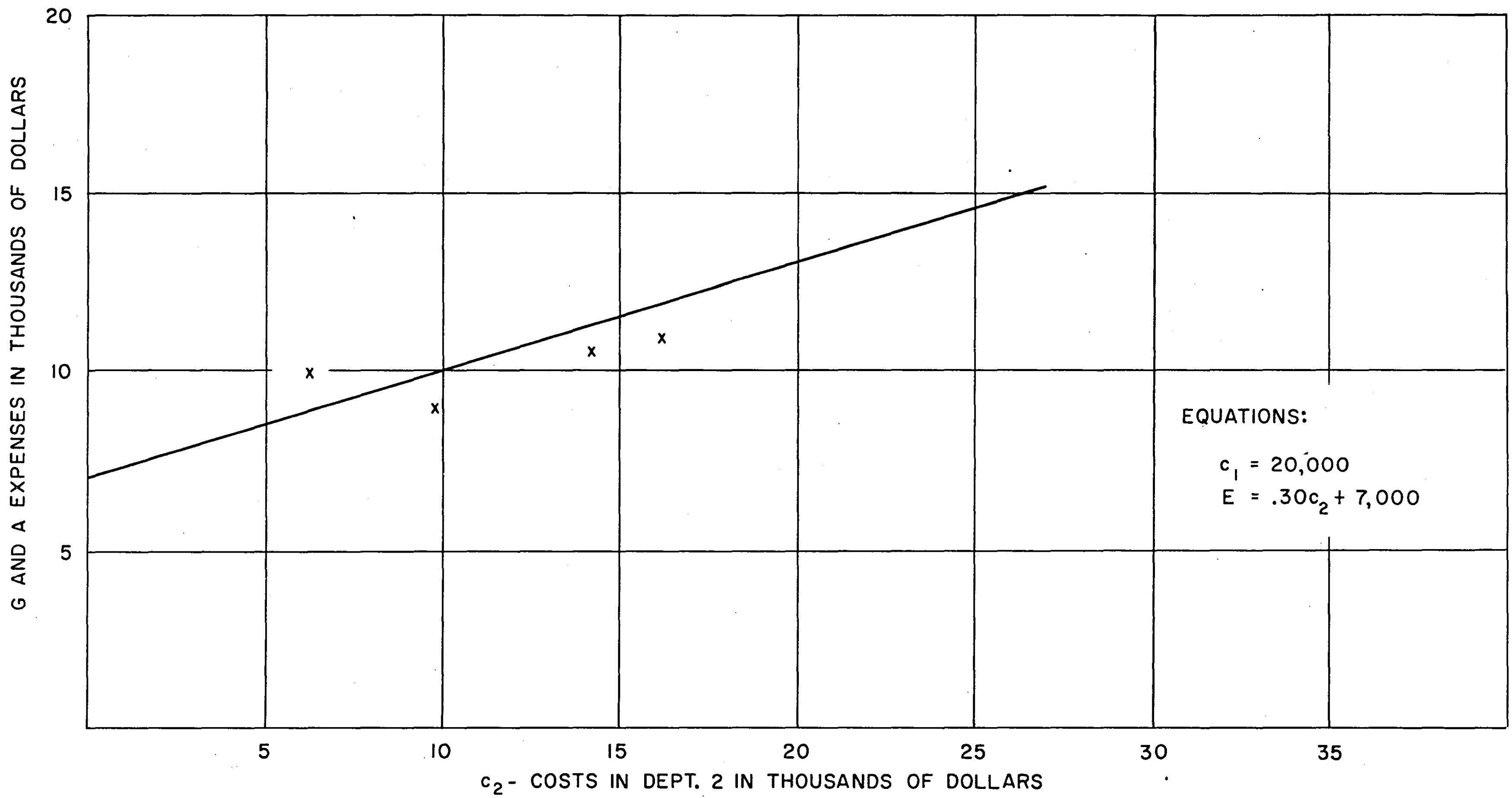


FIG. 10. GENERAL AND ADMINISTRATIVE EXPENSES FOR THE BETA CORPORATION

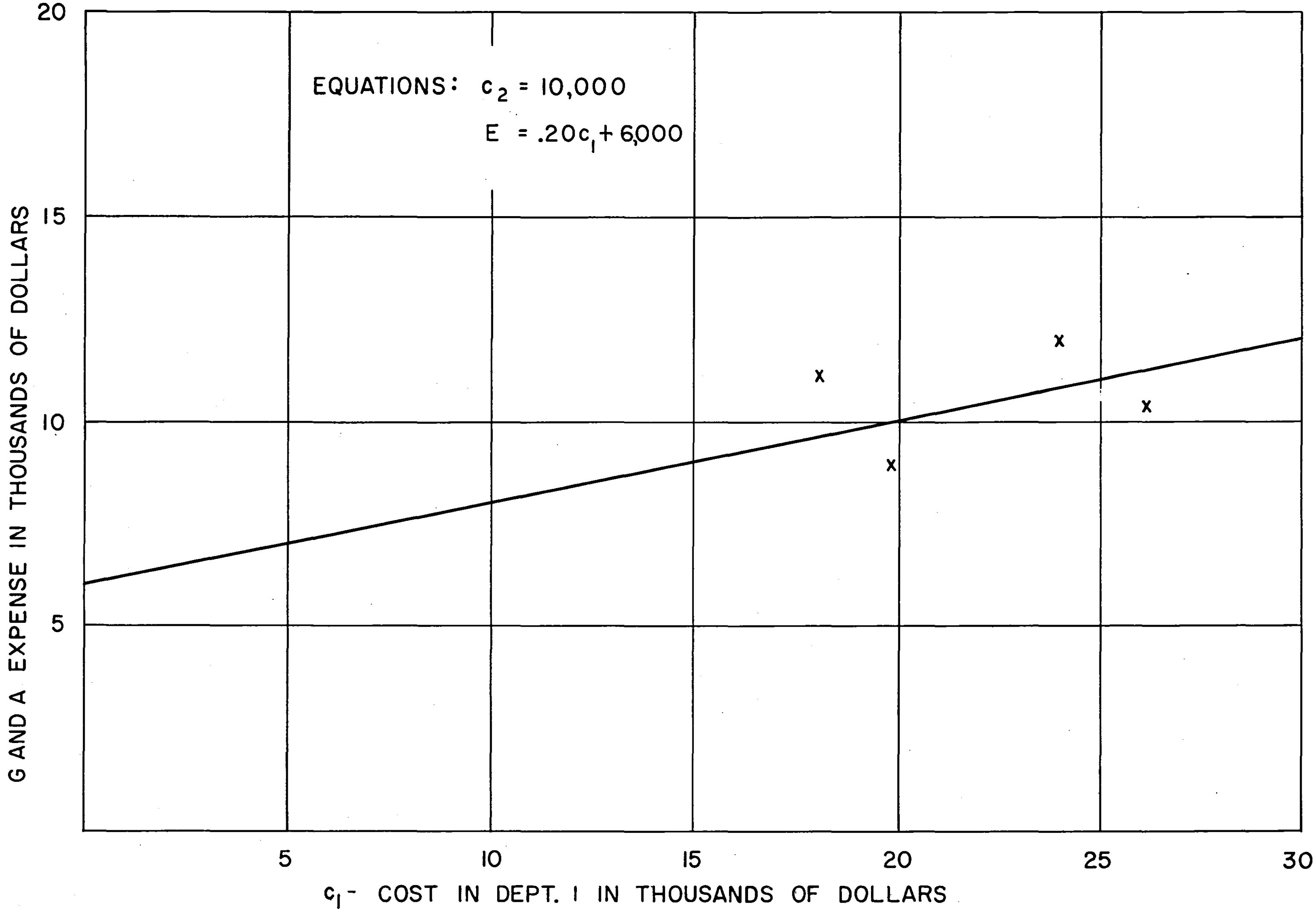


FIG. 11. GENERAL AND ADMINISTRATIVE EXPENSES FOR THE BETA CORPORATION

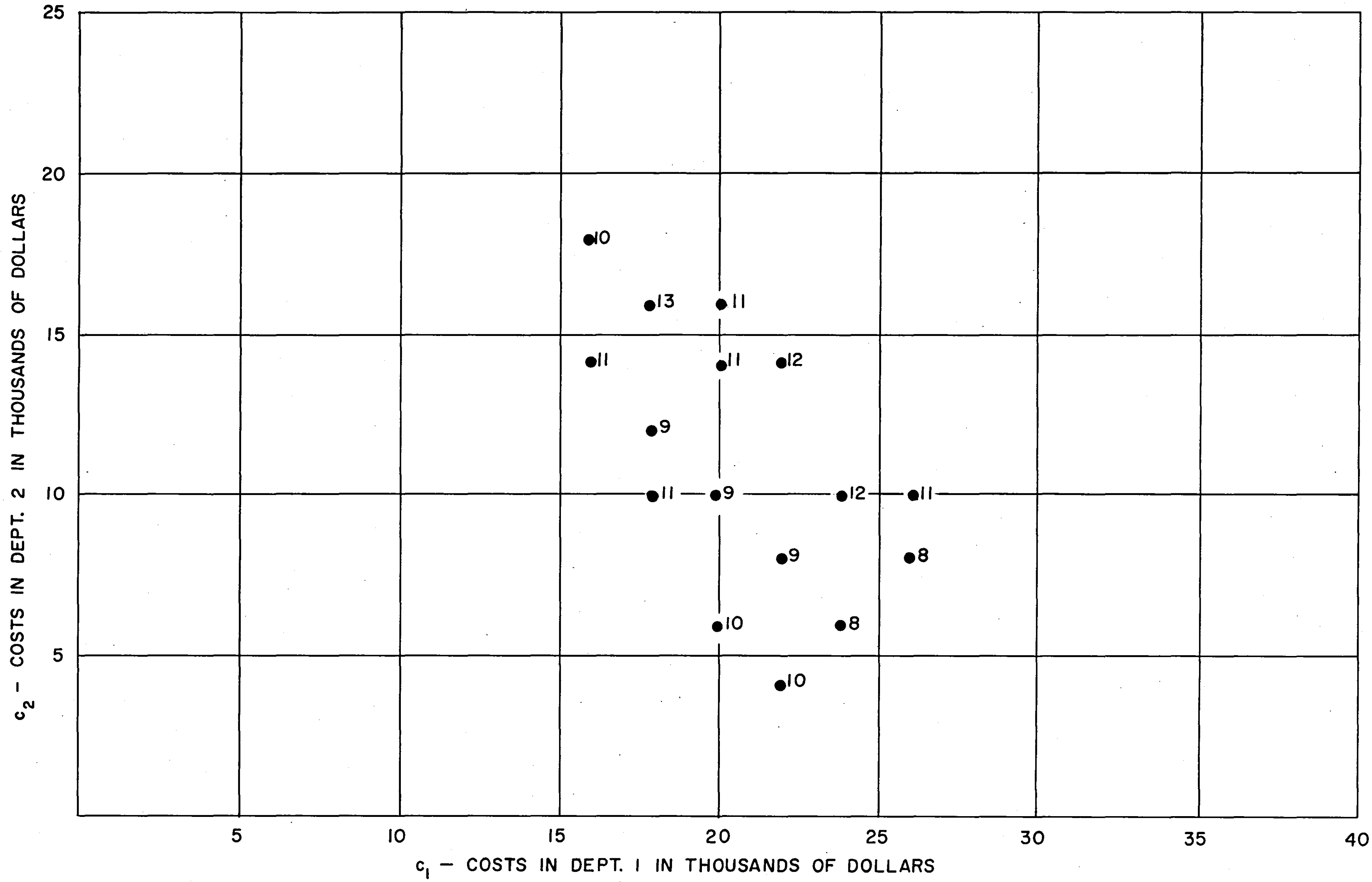


FIG. 12. GENERAL AND ADMINISTRATIVE EXPENSES FOR THE BETA CORPORATION (THE NUMBER SHOWS G AND A EXPENSES IN THOUSANDS OF DOLLARS.)

c_1 = COSTS IN DEPT. 1 c_2 = COSTS IN DEPT. 2 E = G&A EXPENSE

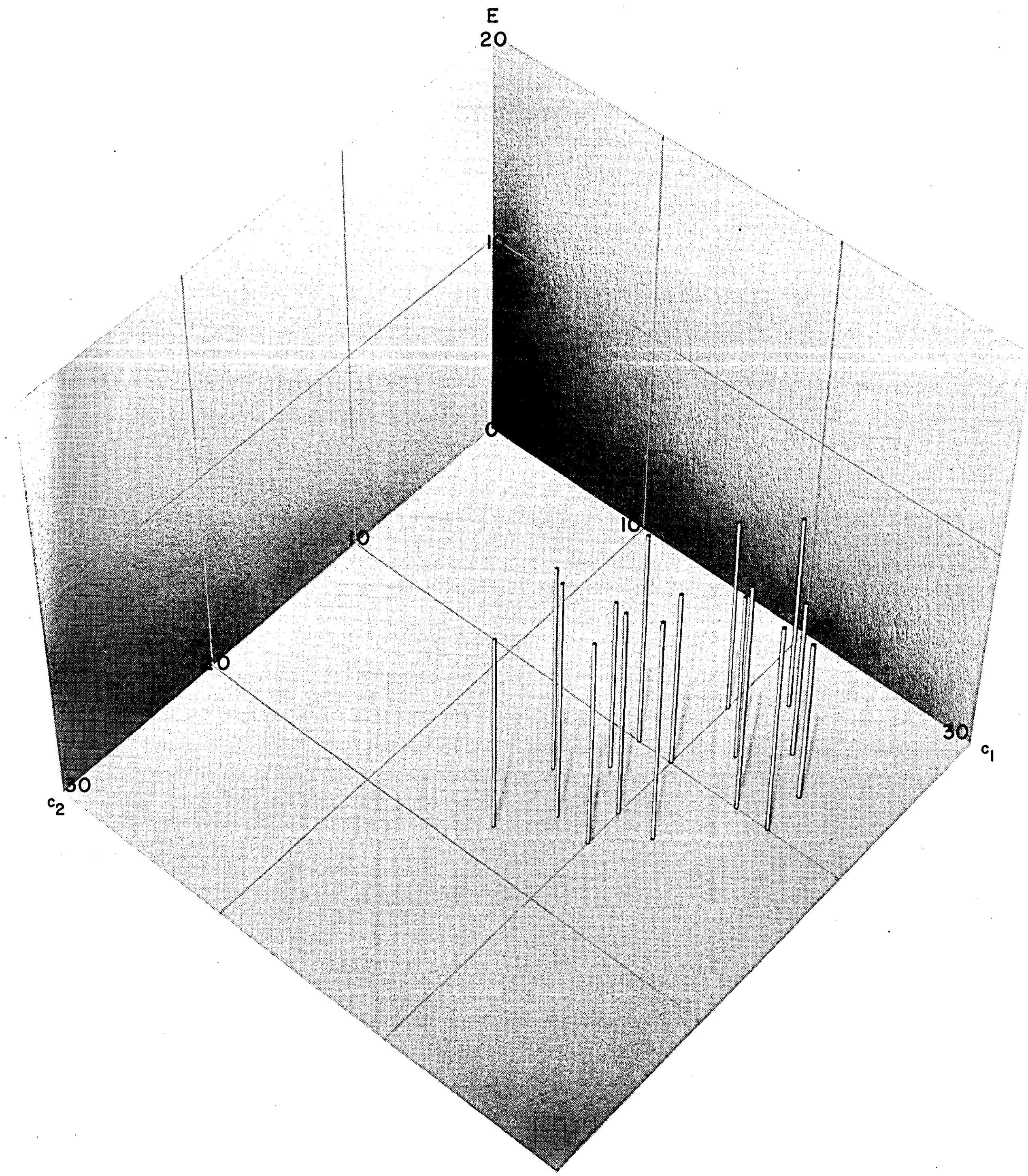


FIG. 13 GENERAL AND ADMINISTRATIVE EXPENSES FOR THE BETA CORPORATION

EQUATION: $E = .20c_1 + .30c_2 + 3000$

c_1 = COSTS IN DEPT. 1 c_2 = COSTS IN DEPT. 2   E = G&A EXPENSES

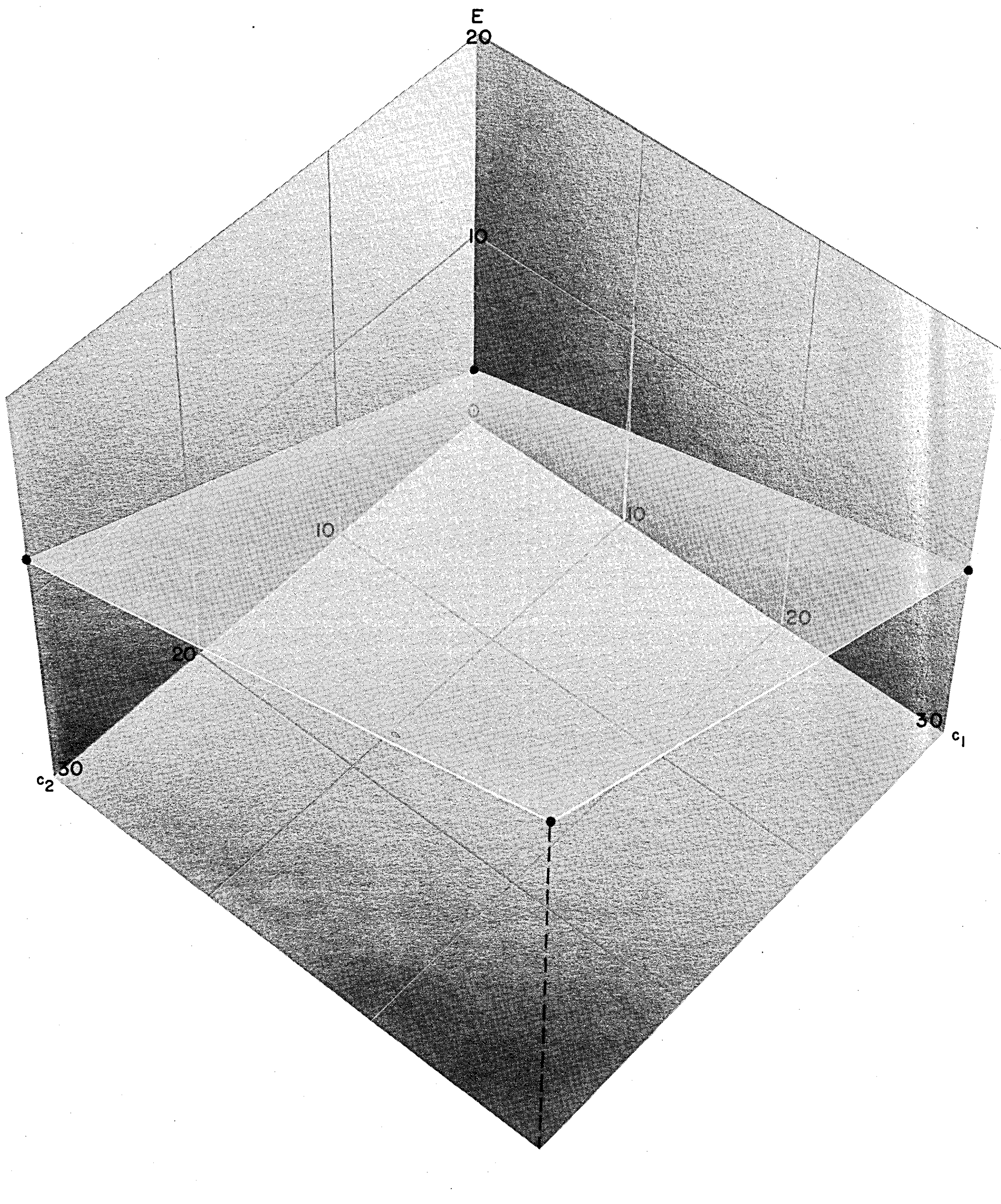


FIG. 14 GENERAL AND ADMINISTRATIVE EXPENSES FOR THE BETA CORPORATION

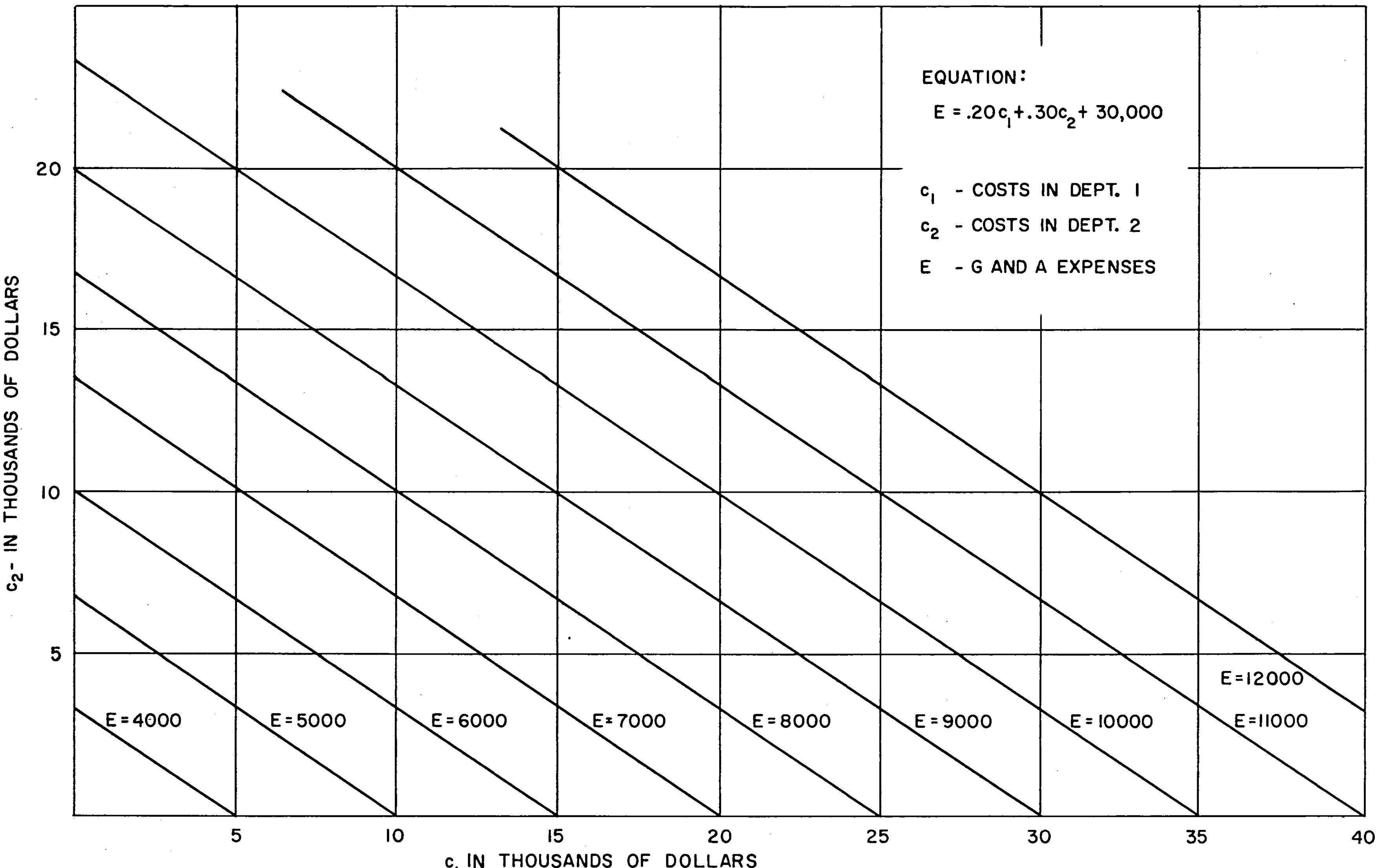


FIG. 15 GENERAL AND ADMINISTRATIVE EXPENSES FOR THE BETA CORPORATION

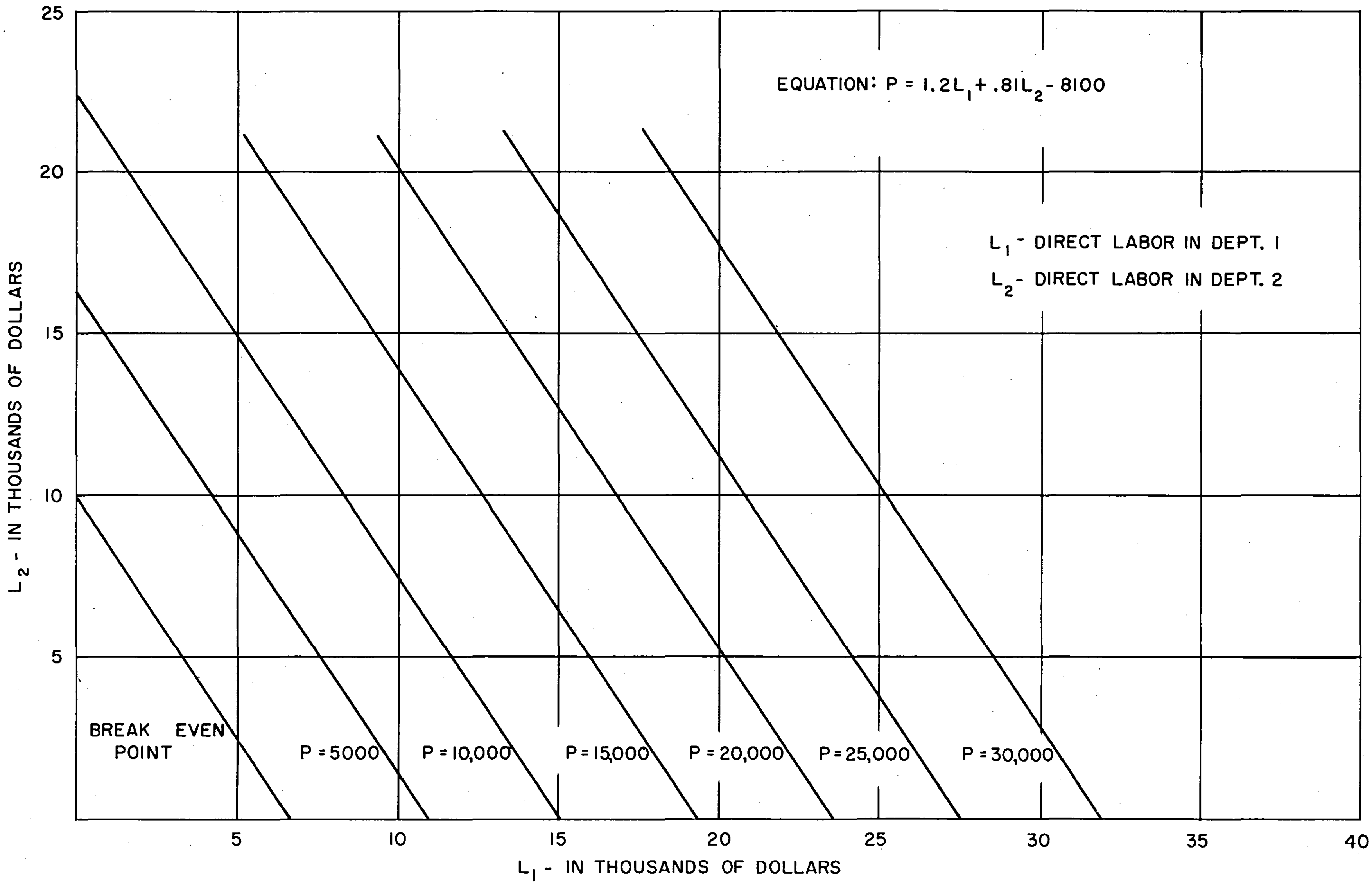


FIG. 16 PROFIT FOR THE BETA CORPORATION

	I	L_1	L_2	e_1	e_2	c_1	c_2	E	R	C
e_1	1000	.5								
c_1		+1		+1						
e_2	3000		.3							
c_2			+1		+1					
E	3000					.2	.3			
C						+1	+1	+1		
R		3	2.5							
P									+1	-1

FIG. 17. COST AND PROFIT TABLE FOR THE BETA CORPORATION.

	I	L_1	L_2	e_1	e_2	c_1	c_2	E	R	C
e_1	a_1	b_1								
c_1		+1		+1						
e_2	a_2		b_2							
c_2			+1		+1					
E	B					A_1	A_2			
C						+1	+1	+1		
R		$1+r_1$	$1+r_2$							
P									+1	-1

FIG. 18. COST AND PROFIT TABLE FOR CORPORATIONS SIMILAR TO THE BETA CORPORATION.

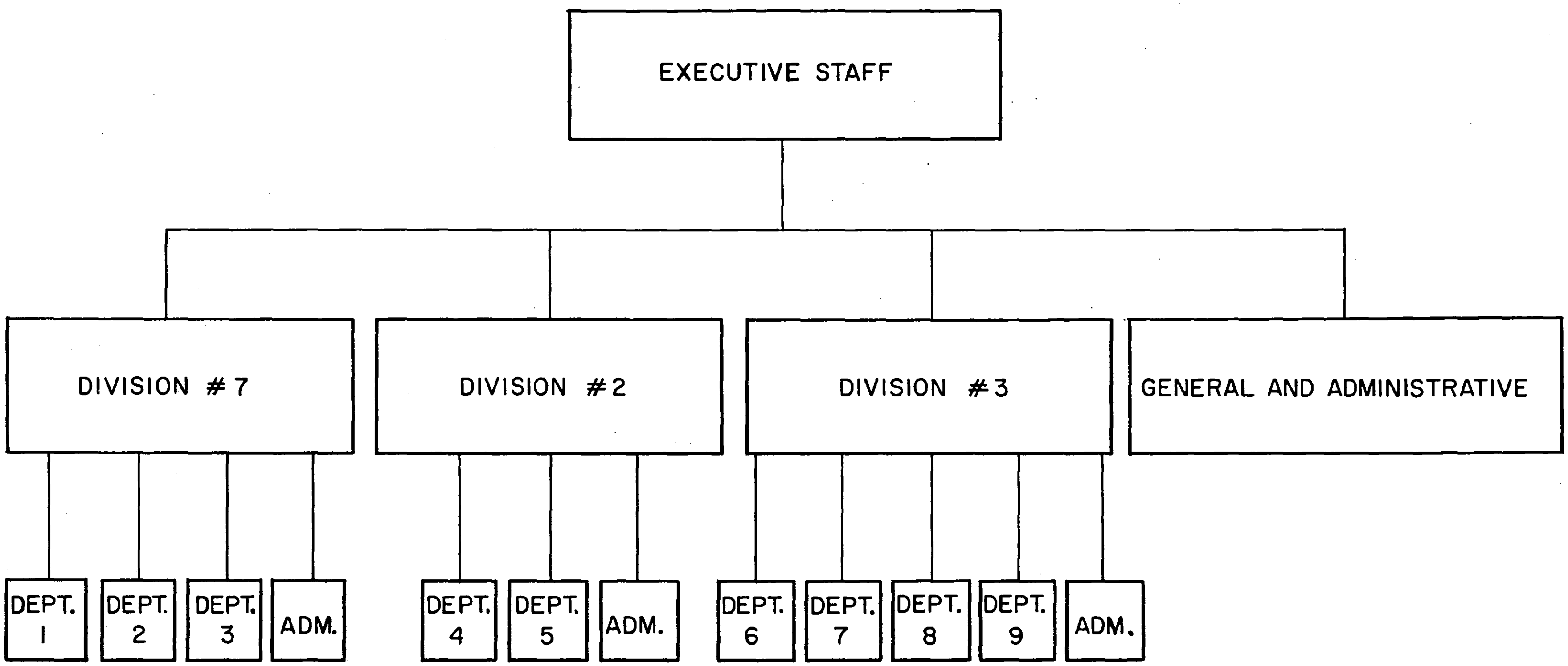


FIG. 19 ORGANIZATION CHART FOR THE GAMMA CORPORATION

	I	L_1	L_2	L_3	e_1	e_2	e_3	c_1	c_2	c_3	E
e_1	a_1	b_1									
c_1		+1			+1						
e_2	a_2		b_2								
c_2			+1			+1					
e_3	a_3			b_3							
c_3				+1			+1				
E_1	B_1							A_1	A_2	A_3	
C_1								+1	+1	+1	+1
R_1		$1+r_1$	$1+r_2$	$1+r_3$							

TABLE FOR DIVISION I

	I	R_1	R_2	R_3	C_1	C_2	C_3	E_c	R_c	C_c
R		+1	+1	+1						
E_c	fo				f_1	f_2	f_3			
C_c					+1	+1	+1	+1		
P_c									+1	−1

TABLE FOR CORPORATE COSTS AND PROFIT

FIG. 20 COST AND PROFIT TABLE FOR THE GAMMA CORPORATION

	DIRECT LABOR	INDIRECT	"OVERHEAD"
JAN.	24,000	16,000	67%
FEB.	23,000	22,000	79%
MAR.	24,000	19,000	79%
APR.	26,000	20,000	77%
MAY	32,000	21,000	66%
JUNE	20,000	17,000	85%
JUL.	26,000	18,000	69%
AUG.	22,000	14,000	64%
SEPT.	28,000	19,000	68%
OCT.	24,000	17,000	71%
NOV.	30,000	18,000	60%
DEC.	22,000	19,000	86%

TABLE I. COST ANALYSIS FOR THE ALPHA CORPORATION

	c_1	c_2	E
1	20	6	10,000
2	20	16	11,000
3	18	12	9,000
4	20	10	9,000
5	22	14	12,000
6	18	16	13,000
7	16	18	10,000
8	24	6	8,000
9	24	10	12,000
10	16	14	11,000
11	26	10	11,000
12	26	8	8,000
13	22	4	10,000
14	18	10	11,000
15	22	8	9,000
16	20	14	11,000

$$E = .20c_1 + .30c_2 + 3000$$

TABLE 2. GENERAL AND ADMINISTRATIVE EXPENSES FOR THE BETA CORPORATION.

	DEPT. 1	DEPT. 2
1	L_1 = 15,000	L_2 = 5,000
2	e_1 = 8,500	e_2 = 4,500
3	c_1 = 23,500	c_2 = 9,500
4	$.20c_1$ 4,700	$.30c_2$ = 2,850
5	$\frac{3000}{c_1+c_2}c_1$ = 2,140	$\frac{3000}{c_1+c_2}c_2$ = 860
6	E'_1 = 6,840	E'_2 = 3,710
7	C'_1 = 30,340	C'_2 = 13,210
8	R_1 = 45,000	R_2 = 12,500
9	P'_1 = 14,660	P'_2 = −710

$$P_c = 1.2L_1 + .81L_2 - 8100 = 13,950$$

$$E = .20c_1 + .30c_2 + 3000 = 10,550$$

$$E'_1 = .20c_1 + \frac{3000}{c_1 + c_2} c_1$$

$$E'_2 = .30c_2 + \frac{3000}{c_1 + c_2} c_2$$

TABLE 3. DEPARTMENTAL COSTS AND PROFITS IN THE BETA CORPORATION

	DEPT. 1		DEPT. 2	
1	L_1 =	30,000	L_2 =	5,000
2	e_1 =	16,000	e_2 =	4,500
3	c_1 =	46,000	c_2 =	9,500
4	$.20c_1$ =	9,000	$.30c_2$ =	2,850
5	$\frac{3000}{c_1+c_2}c_1$ =	2,500	$\frac{3000}{c_1+c_2}c_2$ =	500
6	E'_1 =	11,700	E'_2 =	3,350
7	C'_1 =	57,700	C_2 =	12,850
8	R_1 =	90,000	R_2 =	12,500
9	P'_1 =	32,300	P'_2 =	− 350

P_c = 31,950

TABLE 4. DEPARTMENTAL COSTS AND PROFITS IN THE BETA CORPORATION