

RECOMP II USERS' PROGRAM NO. 1124

PROGRAM TITLE: AGC-90C MODAL ANALYSIS
PROGRAM CLASSIFICATION: Engineering Applications
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PURPOSE: This program is used to compute
natural frequencies and mode shapes
for straight non-uniform beams
represented by up to 30 bays.
DATE: 20 November 1961

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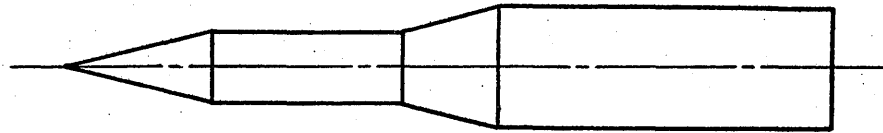
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TITLE: AGC-90C MODAL ANALYSIS

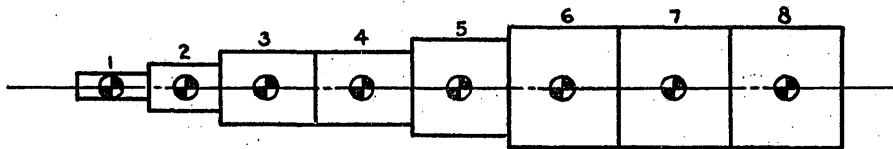
LOCATIONS USED: 0000-0003, 0010-0147, 0530-0767, 2166-2207, 2310-2667, 3000-4577, 4724-4736 and 6742-6777 are used for commands, constants and certain much used intermediate results. Basic data is stored at 7000-7455. Processed data is stored at 5000-6741. Intermediate results are stored at 4600-4723 and 4740-4777. The data location limits are for a maximum sized case of 30 bays. The command location limits include 6 subroutines as follows:

AN-002	Fixed Point Input	0010-0147
AN-014	Floating Point Output	0530-0767
RP-026	Tape Punch	2166-2207
AN-007	Var. Form. Flt. Pt. Input	2310-2507
AGC-85	Fix. Pt. Whole No. Output	4400-4477
AGC-189	Null Search	4500-4577

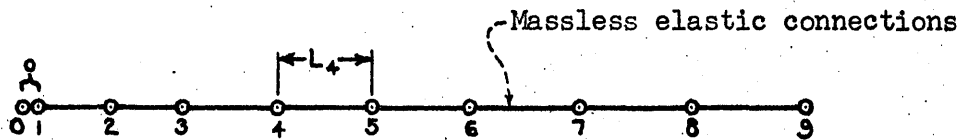
INTRODUCTION: This program assumes that a straight non-uniform beam can be represented by a series of 30 (or less) bays each of which is a uniform beam. Station points are assumed at the ends of each bay, with a double station point (0 and 1) at the outer end of the first bay. Half the mass properties of each bay are associated with the station at each end of the bay. Essentially, the problem must be set up as a series of elastically interconnected point masses (or station points). The point masses can have both linear inertia (MASS) and rotary inertia (mass moment of inertia, MASS I). See Fig. 1.



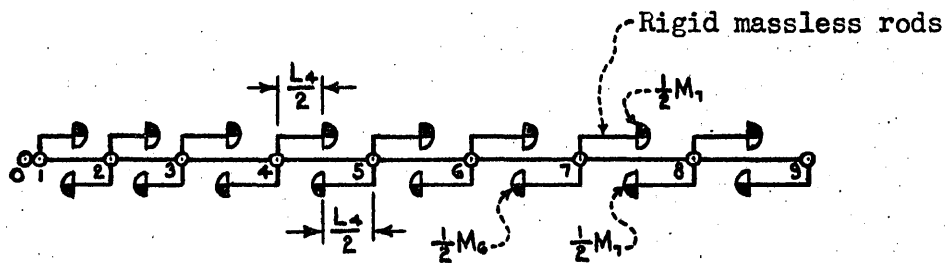
A non-uniform beam.



Above beam represented by 8 bays, each of which is a uniform beam.



Station points for above representation.
 (Note: Sta. 0 & 1 are always 0 distance apart)



How the mass of each bay is associated with the station points.

Figure 1

METHOD:

Each station point is permitted 2 degrees of freedom: lateral translation, H, and pitch, θ . Shear is S and moment is M. When the structure being analyzed is vibrating at frequency ω , we can describe the conditions at some station point i, by a 4 X 1 column vector--

$$\begin{bmatrix} S \\ M \\ H \\ \theta \end{bmatrix}_i$$

It is possible to relate the forces and deflections at station i + 1 to those at station i by a 4 x 4 matrix A_i , thus--

$$\begin{bmatrix} S \\ M \\ H \\ \theta \end{bmatrix}_{i+1} = [A_i] \times \begin{bmatrix} S \\ M \\ H \\ \theta \end{bmatrix}_i$$

This matrix $[A_i]$ is a function of length L, bending stiffness EI, shear stiffness KGA, linear inertia MASS, mass moment of inertia MASS I and frequency in radians per sec. ω . Everything but ω is known. See form of $[A_i]$ below.

$1 + \omega^2(m_{11}b_{11} + m_{12}b_{21})$	$\omega^2(m_{11}b_{12} + m_{12}b_{22})$	ω^2m_{11}	$\omega^2(m_{11}L_i + m_{12})$
$-L_i + \omega^2(m_{21}b_{11} + m_{22}b_{21})$	$1 + \omega^2(m_{21}b_{12} + m_{22}b_{22})$	ω^2m_{21}	$\omega^2(m_{21}L_i + m_{22})$
b_{11}	b_{12}	1	L_i
b_{21}	b_{22}	0	1

Where

$$b_{11} = \frac{L_i^3}{6EI} - \frac{L_i}{KGA}$$

$$b_{12} = -\frac{L_i^2}{2EI}$$

$$b_{21} = \frac{L_i^2}{2EI}$$

$$b_{22} = -\frac{L_i}{EI}$$

$$m_{11} = \frac{M_i + M_{i+1}}{2}$$

$$m_{12} = \frac{M_{i+1}L_{i+1} - M_iL_i}{4}$$

$$m_{21} = \frac{M_{i+1}L_{i+1} - M_iL_i}{4}$$

$$m_{22} = \frac{\text{MASS } I_{i+1} + \text{MASS } I_i}{2}$$

L_i = length of bay i

M_i = mass of bay i

Mass I_i = mass moment of inertia of bay i

EI = bending stiffness of bay i

KGA = shear stiffness of bay i

Continuing the previous line of thought--

$$\begin{Bmatrix} S \\ M \\ H \\ \theta \end{Bmatrix}_1 = [A_0] \begin{Bmatrix} S \\ M \\ H \\ \theta \end{Bmatrix}_0 \qquad \begin{Bmatrix} S \\ M \\ H \\ \theta \end{Bmatrix}_2 = [A_1] \begin{Bmatrix} S \\ M \\ H \\ \theta \end{Bmatrix}_1 = [A_1][A_0] \begin{Bmatrix} S \\ M \\ H \\ \theta \end{Bmatrix}_0$$

$$\begin{Bmatrix} S \\ M \\ H \\ \theta \end{Bmatrix}_R = \prod_{i=0}^{R-1} [A_i] \begin{Bmatrix} S \\ M \\ H \\ \theta \end{Bmatrix}_0 \qquad (\text{Where } R \text{ is the last station no.})$$

Next, we must consider boundary conditions. In general, half of the four quantities S , M , H and θ will be zero at each end of the beam. For example, consider a cantilever beam (FIX-FREE). The deflections, H and θ , will be zero at the fixed end and the forces, S and M , will be zero at the free end.

$$\begin{vmatrix} S \\ M \\ O \\ O \end{vmatrix}_R = \prod_{i=0}^{R-1} [A_i] \begin{vmatrix} O \\ O \\ H \\ \Theta \end{vmatrix}_0$$

Replace

$$\prod_{i=0}^{R-1} [A_i] \quad \text{by} \quad \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$\begin{vmatrix} S \\ M \\ O \\ O \end{vmatrix}_R = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{vmatrix} O \\ O \\ H \\ \Theta \end{vmatrix}_0$$

$$\begin{vmatrix} O \\ O \end{vmatrix}_R = [A_{22}] \begin{vmatrix} H \\ \Theta \end{vmatrix}_0, \text{ (Where } [A_{22}] \text{ is a } 2 \times 2 \text{ matrix.)}$$

This is a set of homogeneous equations. It has a non-trivial solution only if--

$$\text{The determinant of } [A_{22}] = 0.$$

A value of ω which satisfies this equation must be determined. This is done by trial and error. A guess is made for ω and the above determinant is evaluated. The value of the determinant is called "NULL", because we attempt to minimize or nullify it. As various sequential values of ω are tried, the algebraic sign of each NULL is noted. When this sign changes, it indicates that the NULL vs. ω curve has crossed the ω axis. Linear interpolation is used to converge on the ω at the crossing point.

Next, setting $H = 1$, O can be computed, thus establishing

$$\begin{vmatrix} S \\ M \\ H \\ \Theta \end{vmatrix}_0 \equiv \begin{vmatrix} O \\ O \\ 1 \\ \Theta \end{vmatrix}_0$$

Now, since ω is known, all the A_i 's are known and forces and deflections can be computed for the rest of the stations. The ω as determined in this way is one of the "natural frequencies" of the system (in radians/sec.) and the forces and deflections computed using it represent the mode shape for that frequency.

In the above, note that the 2×2 matrix selected from $\prod A_i$ is a function of the boundary conditions.

The rows correspond to zero elements in

$$\begin{vmatrix} S \\ M \\ H \\ \Theta \end{vmatrix}_R ,$$

and the columns correspond to non-zero elements in

$$\begin{vmatrix} S \\ M \\ H \\ \Theta \end{vmatrix}_0 .$$

When solving the system of homogeneous equations, one of the forces or deflections (unknowns) was set = 1. The element in the row and column corresponding to the other force or deflection is called "CHECK". It has been found by practical experience that for a satisfactory mode shape, "CHECK" should be at least 10^6 x "NULL".

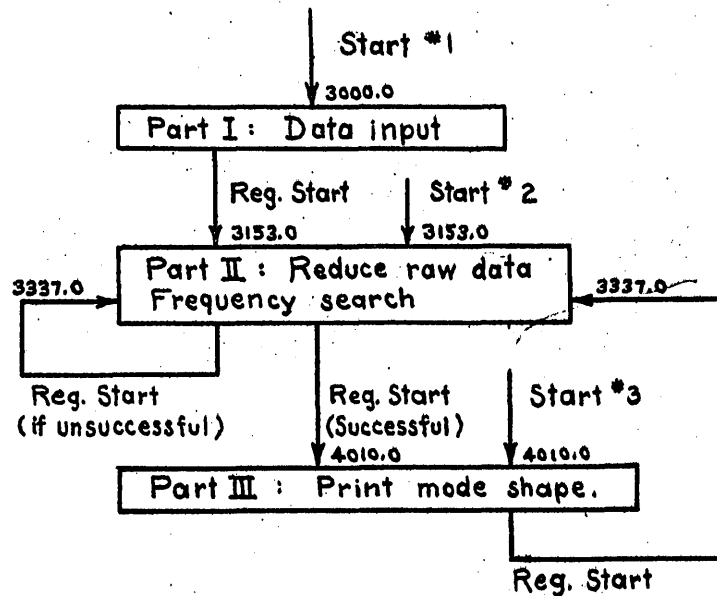
RESTRICTIONS: The program will handle structures which can be represented by a maximum of 30 bays, each of which is a uniform beam with its C. G. at its center. (See Fig. 1 and sample problem 2). Boundary conditions are restricted to the following set:

<u>Sense Switches On (down)</u>	<u>Boundary Conditions</u>
D	FIX-FIX
C	FIX-PIN
CD	FIX-FREE
B	PIN-PIN
B D	PIN-FREE
BC	FREE-FREE

The highest numbered station goes with the first word of the boundary condition pair. Thus, for a 10 bay beam run with FIX-PIN boundary conditions, station 0 has a pin connection and station 11 is fixed. The highest bending mode that can be obtained with any kind of accuracy depends on the number of bays used in the representation. As a rough "rule of thumb", it is suggested that the highest mode attempted be less than a third of the number of bays. Thus, for a 30 bay representation, the first 3 or 4 modes should be pretty good but the 10th one will probably be rather approximate. To give the beginner at modal analysis some feeling for what is "good", if an experimental and computed frequency agree within 1%, we're delighted. If they disagree as much as 10% we're appalled, and start looking for a mistake in the data or at least an explanation. Usually the lower modes are more accurate than the higher ones.

USAGE:

For operating purposes, the various starts are summarized graphically below



1. Set left margin at 10 and tab stops at 16, 30, 46, 62 and 78.
2. Read in program tape.
3. Set SSB and SSC according to the desired data input format as shown on the next page. (The program will stay in any one format unless the sense switches are changed. Formats can be mixed. Sample problem 2 uses all 4 formats. To enter the next bay in a different format, change the sense switches and press start #1.)

SSB SSC

DATA INPUT FORMAT

OFF OFF

E 00000
G 00000

BAY	L	D	T	MASS	EFF. D
10	00000	00000	00000	00000	00000
11	00000	00000	00000	00000	00000
:	:	:	:	:	:

OFF ON

E 00000
G 00000

BAY	L	D	T	MASS	MASS I
15	00000	00000	00000	00000	00000
16	00000	00000	00000	00000	00000
:	:	:	:	:	:

ON OFF

BAY L
20 00000
21 00000

EI	KGA	MASS	EFF. D
00000	00000	00000	00000
00000	00000	00000	00000
:	:	:	:

ON ON

BAY L
25 00000
26 00000

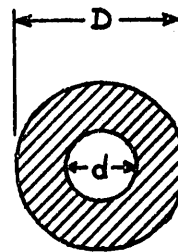
EI	KGA	MASS	MASS I
00000	00000	00000	00000
00000	00000	00000	00000
:	:	:	:

4. After the sense switches have been set, press start #1. The appropriate headings will be printed out. Enter E, G, EFF.D and MASS I with carriage returns. Enter BAY, L, EI, KGA, D, T and MASS with tabs. All data except BAY are entered in ABC07 format. BAY is an integer in ANCO2 format.

The headings have meanings as follows:

- E = Young's modulus of elasticity, lb./in.²
- G = Shearing modulus of elasticity, lb./in.²
- BAY = Bay number. $1 \leq (\text{An integer}) \leq 30$
- L = Length of the bay, in.
- D = Diameter of the bay (if circular), in.
- T = Thickness of bay (if thin circular shell), in.
- MASS = Mass of bay, lb. sec²/in.
- EFF.D = A diameter used to calculate the mass moment of inertia of a cylindrical bay about one end using the formula for a homogeneous solid cylinder. If the cylinder has a concentric hole of diameter d and the outside diameter is D , then $\frac{\text{EFF.D}}{D}$ as a function of $\frac{d}{D}$ can be obtained from the following table:

$\frac{d}{D}$	$\frac{\text{EFF.D}}{D}$
=	=====
0.0	1.000
0.1	1.005
0.2	1.020
0.3	1.044
0.4	1.077
0.5	1.118
0.6	1.166
0.7	1.221
0.8	1.281
0.9	1.345
1.0	1.414



- EI = Bending stiffness (or average bending stiffness of a non-uniform bay), lb. in.². E is as defined above and I is second moment of area.
- KGA = Shear stiffness (or average shear stiffness of a non-uniform bay), lb. K is a constant depending on the cross section (0.5 for thin circular shell) G is as defined above and A is cross section area.
- MASS I = The mass moment of inertia of the bay around one end, in. lb. sec².

5. Bays can be entered in any order and changed at any time without affecting the other bays. After the last bay has been entered, type "99" under BAY. This causes 0.9×10^{10}

to be stored as a flag at the end of the data. (Actually, any number ≥ 31 can be used, but for the sake of uniformity and to avoid confusion with bay numbers, we always use 99) If a bay in the middle of the structure is changed later, do not enter 99 again, since that would cut the structure off right after the new bay.

6. After the data has been entered, and the paper adjusted (if necessary) sense switches should be set to establish the boundary conditions. See the list under "RESTRICTIONS". The operator can then press either the regular start or start #2. Only start #2 could be used if the data had been previously punched and re-entered on the photo-reader. The machine will spend a minute or so reducing the raw data and re-storing it in a different location.

7. The program will then ask for INITIAL F, DELTA F and FINAL F.

INITIAL F = The first frequency, f_0 , in c.p.s. that we want to try. Normally, we try to select this so that it is less than the desired natural frequency.

DELTA F = The frequency increment, Δf , that we want to use in the search. Thus the program will try in sequence f_0 , $f_0 + \Delta f$, $f_0 + 2\Delta f$, $f_0 + 3\Delta f$, etc. (Unless of course a crossing is found between any of these).

FINAL F = An upper limit for the frequencies that we want the machine to try. Normally we try to select this somewhat higher than the desired natural frequency. The interval (FINAL F - INITIAL F) does not have to be an exact multiple of DELTA F, in which case FINAL F would possibly not be tried.

Enter these 3 quantities in ANOC7 format using carriage returns.

8. The program will print out F, C.P.S., NULL and (Bound. Cond.) If the operator forgets to set the sense switches (all off) the machine will halt. Set switches and press the regular start button. The program will then proceed to compute NULL's for the series of frequencies specified. When a sign change occurs in the NULL's, the computer will start to converge on the crossing frequency using linear interpolation. For all the beams that the author has tried, the algebraic signs of the nulls, below the first frequency, were as follows:

FIX-FIX	+
FIX-PIN	-
FIX-FREE	+
PIN-PIN	-
PIN-FREE	-
FREE-FREE	+

This can often serve as a useful guide when searching for a specific frequency. For example, suppose one is looking for the 3rd PIN-FREE frequency of a beam. If the operator thinks that the 3rd frequency is about 20 c.p.s., he might put in 18 for INITIAL F. Suppose that the first NULL comes out positive. This would tell him that he's probably too high, and should start his search somewhat lower. Of course he also might really be off in his estimate, and be between the 1st and 2nd frequency. See Figure 2 below.

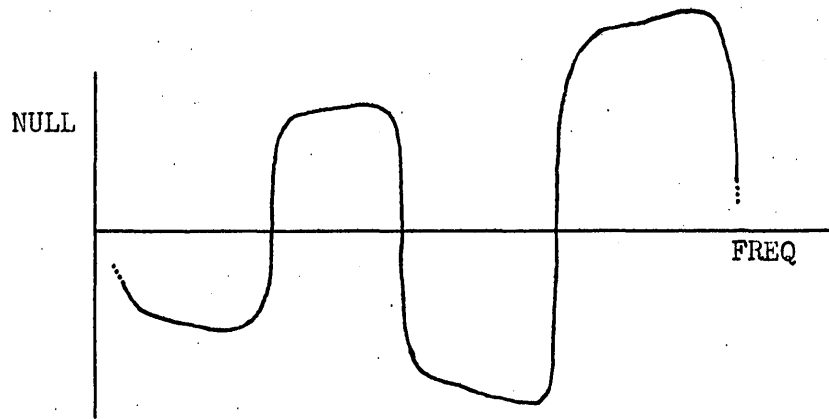


Fig. 2 PIN-FREE NULL plot

Caution: Sometimes, if f is too coarse relative to the distance between natural frequencies, convergence may be quite slow. In this case, depress the stop button, then start #2. Enter new INITIAL F, DELTA F, (smaller than the old one) and FINAL F. Instead of pressing start #2 above, to avoid reducing the raw data again, one can manually transfer to location 3337.0. Start #2 was not set up to go to 3337.0, to avoid the possibility of accidentally trying to run a new case with unreduced data.)

The program decides that the convergence is satisfactory when NULL becomes zero, or when 2 succeeding values of F differ by less than 10^{-11} F. This means that sometimes several F's are printed which look identical. They really vary slightly in the last few binary bits, but still convert to the same decimal number. Also, the NULL's sometimes behave a little erratically near the end. This effect is probably due to a certain randomness in the way the numbers round off. The frequencies are not carried out to 11 figures with any idea that they are that accurate. The object is to get a good match on the boundary conditions. In general, anything beyond the first 3 or 4 significant figures of the frequency are meaningless. When a satisfactory convergence

is attained, the machine carriage returns 3 times and stops.

9. To print the mode shape, adjust the paper (if necessary) and press either the regular start or start #3. Start #3 could be used if the operator wanted an approximate mode shape printed out before full convergence was attained.
10. After the mode shape has been printed, and the paper adjusted (if necessary) pressing the regular start will enable a new frequency search to be started without reducing the raw data again. If start #2 is pressed, this reduction will be made, wasting a little time, although it doesn't hurt anything. This reduction of raw data can also be eliminated by manually transferring to location 3337.0.

If the computer somehow transfers to location 0000, it will stop at 0000.1 with the address of the place it transferred from in the accumulator @ 38.

11. If it is desired to save the data for future use, it can be punched out. The following table give storage location limits for all possible cases. In each case, the last 2 words should be the flag, 0.9×10^{99} . In instruction format this is +64 5263.1-45 3702.0, +00 0000.0-00 0244.1.

No. of Bays	Data Including Flag
1	7000-7013
2	7000-7025
3	7000-7037
4	7000-7051
5	7000-7063
6	7000-7075
7	7000-7107
8	7000-7121
9	7000-7133
10	7000-7145
11	7000-7157
12	7000-7171
13	7000-7203
14	7000-7215
15	7000-7227
16	7000-7241
17	7000-7253
18	7000-7265
19	7000-7277
20	7000-7311
21	7000-7323
22	7000-7335
23	7000-7347
24	7000-7361
25	7000-7373
26	7000-7405
27	7000-7417
28	7000-7431
29	7000-7443
30	7000-7455

To use such punched data, read in on photo-reader and press start #2. We punch out the "raw data" rather than the "reduced data" because the latter is over 3x as long as the former, and the extra punching time would be longer than the time required to reduce it internally.

12. For each boundary condition, there is an alternative way of normalizing the answers. This alternative normalization can be achieved by manually changing commands as follows:

<p>FIX-FIX, Normalized on M</p> <p>Location Command</p> <p>4030 +34 4762.0 +05 4760.0</p> <p>4031 +35 4740.0 +30 3154.0</p> <p>4032 +35 4742.0 +30 3170.0</p>	<p>FIX-PIN, Normalized on θ</p> <p>Location Command</p> <p>4035 +57 4074.0 +34 4766.0</p> <p>4036 +05 4760.0 +35 4740.0</p> <p>4037 +30 3154.0 +66 4040.0</p> <p>4040 +35 4746.0 +30 3170.0</p>
<p>FIX-FREE, Normalized on θ</p> <p>Location Command</p> <p>4043 +57 4074.0 +34 4766.0</p> <p>4044 +05 4764.0 +35 4744.0</p> <p>4045 +30 3154.0 +35 4746.0</p>	<p>PIN-PIN, Normalized on θ</p> <p>Location Command</p> <p>4052 +34 4756.0 +05 4750.0</p> <p>4053 +35 4740.0 +30 3154.0</p> <p>4054 +35 4746.0 +30 3170.0</p>
<p>PIN-FREE, Normalized on θ</p> <p>Location Command</p> <p>4060 +34 4756.0 +05 4754.0</p> <p>4061 +35 4744.0 +30 3154.0</p> <p>4062 +35 4746.0 +30 3170.0</p>	<p>FREE-FREE, Normalized on θ</p> <p>Location Command</p> <p>4065 +57 4074.0 +34 4746.0</p> <p>4066 +05 4744.0 +35 4744.0</p> <p>4067 +64 4070.0 +57 7760.0</p> <p>4070 +30 3154.0 +35 4746.0</p>

CODING

INFORMATION:

18 @ 39	3130	
2 @ 38	3131	
310 @ 38	3132	
(6766) ₈ @ 38	3133	
10 @ 20	3134	
$\pi/64$	3135-3136	
$-\pi/4$	3137-3140	
0.75	3141-3142	
-2.0	3143-3144	
4.0	3145-3146	
12.0	3147-3150	
0.9×10^{99}	3151-3152	
1.0	3154-3155	
(7000) ₈ @ 18	3156	
(5040) ₈ @ 18	3157	
4 flt. pt. zero's	3170-3177	
8 @ 18	3274	
10 @ 18	3275	
20 @ 18	3276	
32 @ 18	3277	
2 @ 18, 2 @ 38	3300	
-3.0	3302-3303	
0.5×10^{99}	3304-3305	
2π	3324-3325	
(5040) ₈ @ 18	3436	
8 @ 18	3437	
0.5×10^{99}	3446-3447	
1×10^{-11}	3714-3715	
(5000) ₈ @ 18	3777	
2.0	4376-4377	
$\pi/64E$	4700-4701	} Used in Part I
$-\pi/4G, 2\pi f$	4702-4703	
12K @ 38	4704	
(K = Bay No.)		
D	4706-4707	
-2T	4710-4711	
EFF.D	4712-4713	
L, $4L^2$	4714-4715	
$M/12$	4716-4717	
$D-2T, (D-2T)^2, (D-2T)^4$	4720-4721	
D^2	4722-4723	} Used in Part II
F	4706-4707	
$F_i - F_{i-1}$	4710-4711	} Used in Part III
Sta. counter @ 39	4700	
coord. Σ	4702-4703	
Check	4704-4705	
Mtx. Loc. A	4600-4637	
Mtx. Loc. B	4640-4677	
Mtx. Loc. C	4740-4776	
Reduced Data	5000-6741	
Raw Data	7000-7455	

TIME:

Time for Part I, DATA INPUT, is a function of operator skill. If the input format isn't changed and the data doesn't have an absurd number of significant figures, an operator can easily enter 2 or 3 bays per minute.

On Part II, FREQUENCY SEARCH, the time to convergence depends on how well the INITIAL F and DELTA F were chosen. Any one frequency depends on the number of bays in the case. A 30 bay case requires about $1^m 50^s$ for each NULL computed. Convergence should be achieved in less than 15 or 20 lines after a crossing.

For Part III, PRINT MODE SHAPE, the time should be roughly proportional to the number of bays. A 30 bay case requires $10^m 36^s$ to print out a mode shape.

REFERENCES:

Introduction to the Study of Aircraft Vibration and Flutter by Robert H. Scanlan and Robert Rosenbaum, Macmillan Co., 1951.

Fundamentals of Vibration Analysis by N. O. Myklestad. McGraw Hill Book Co., 1956.

EXAMPLE 1:

Given a steel tube as shown below in Fig. 3, Compute the first mode for all 6 boundary conditions using a 10 bay representation.

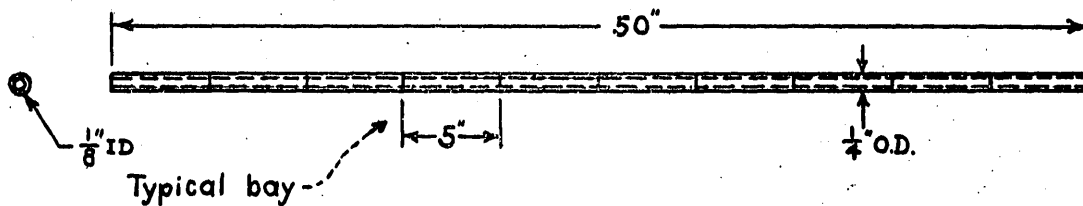
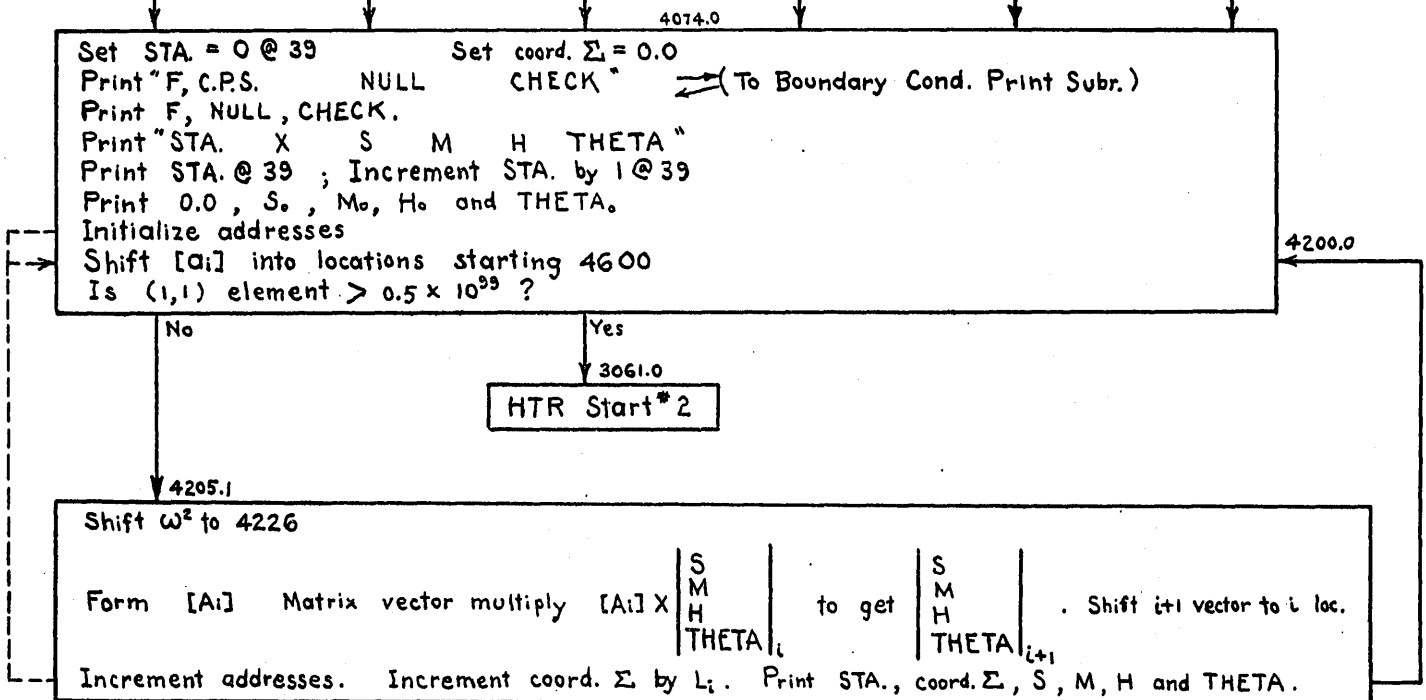
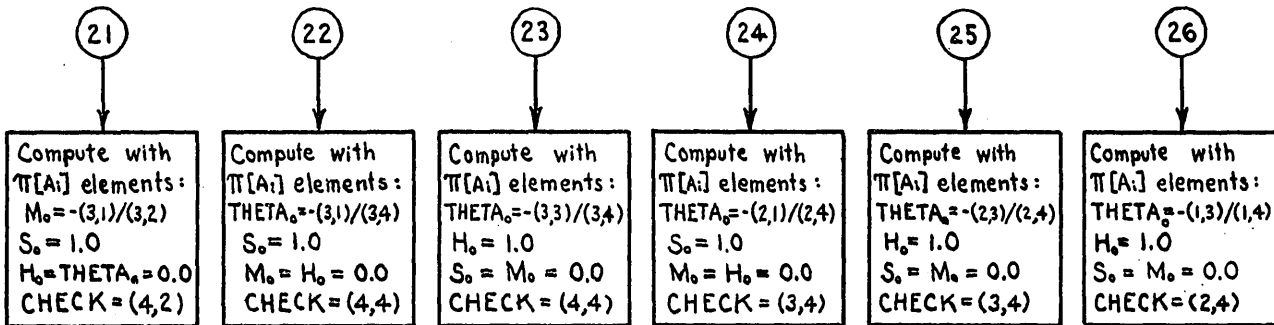
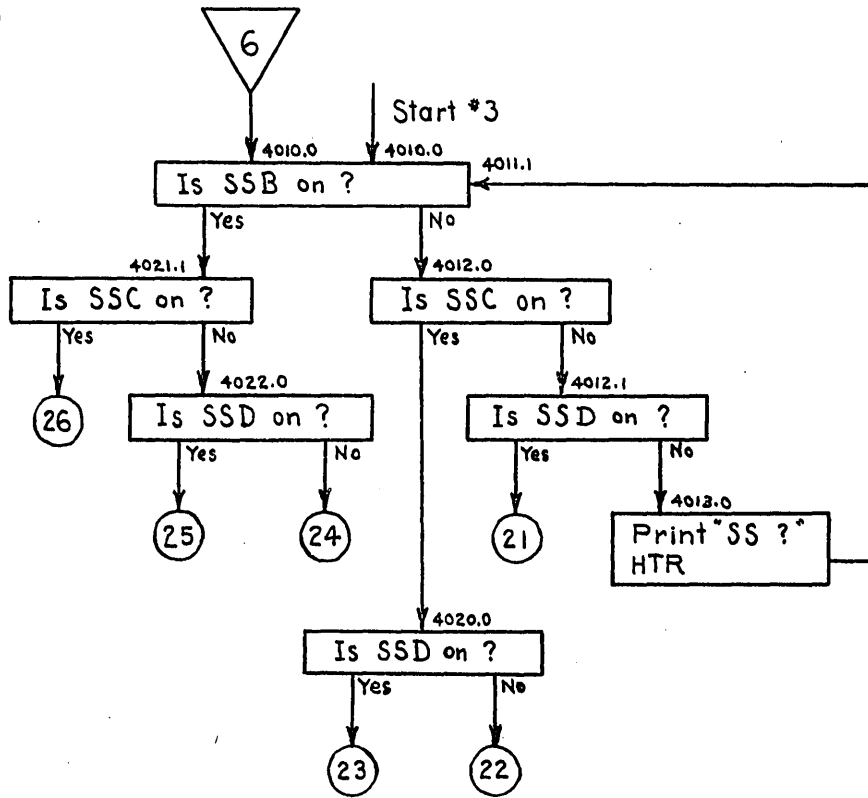


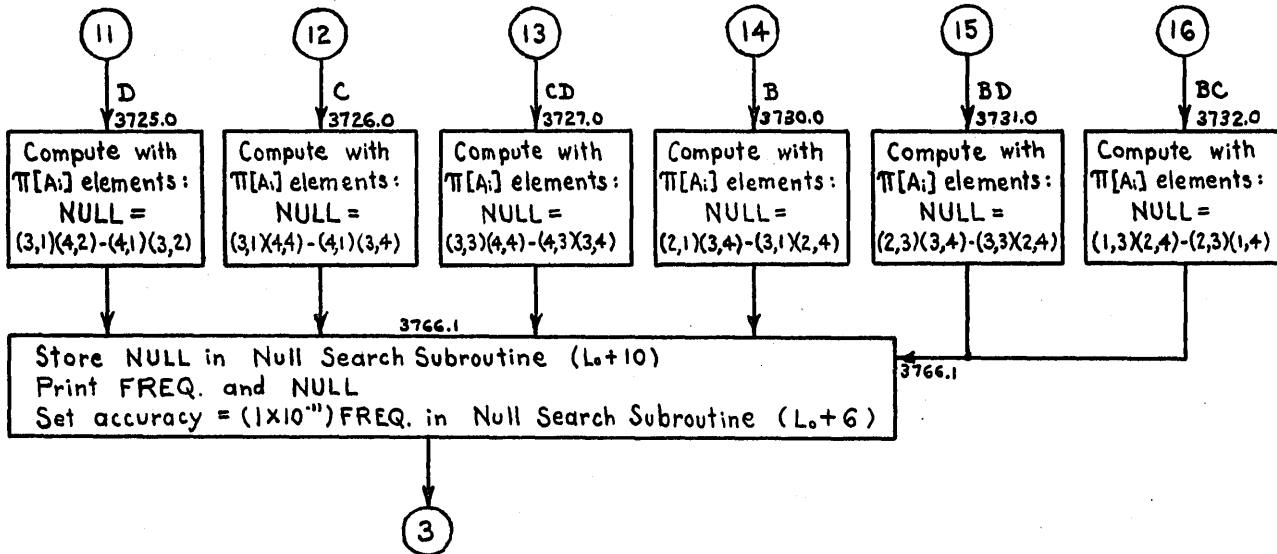
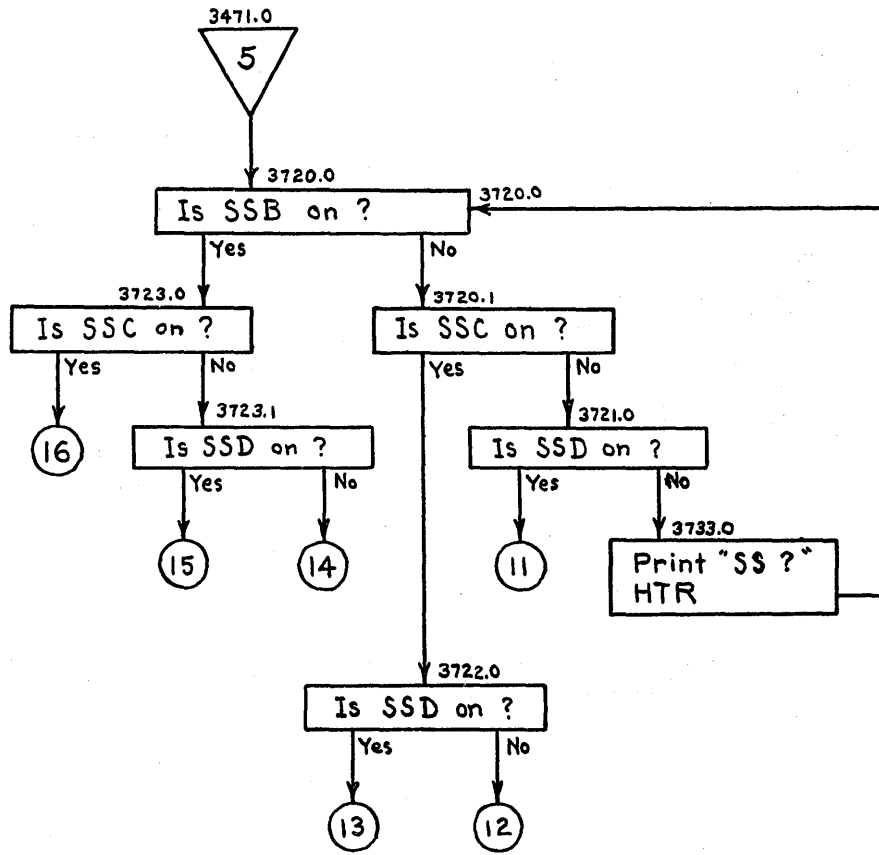
Fig. 3

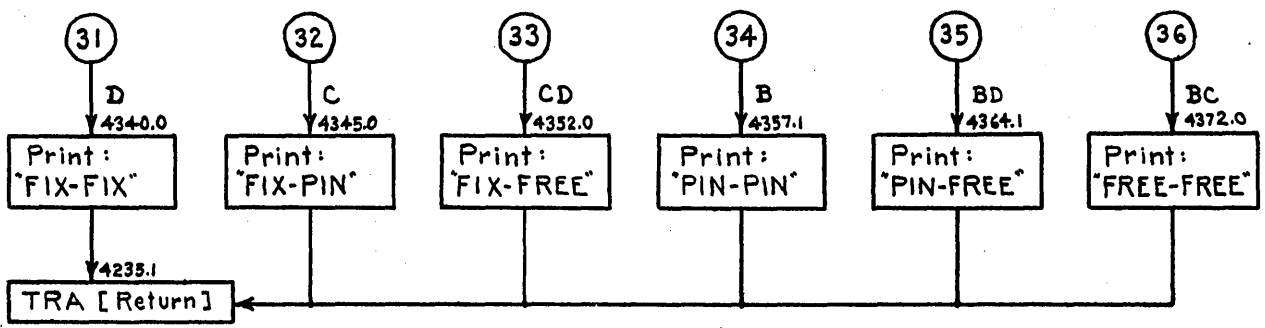
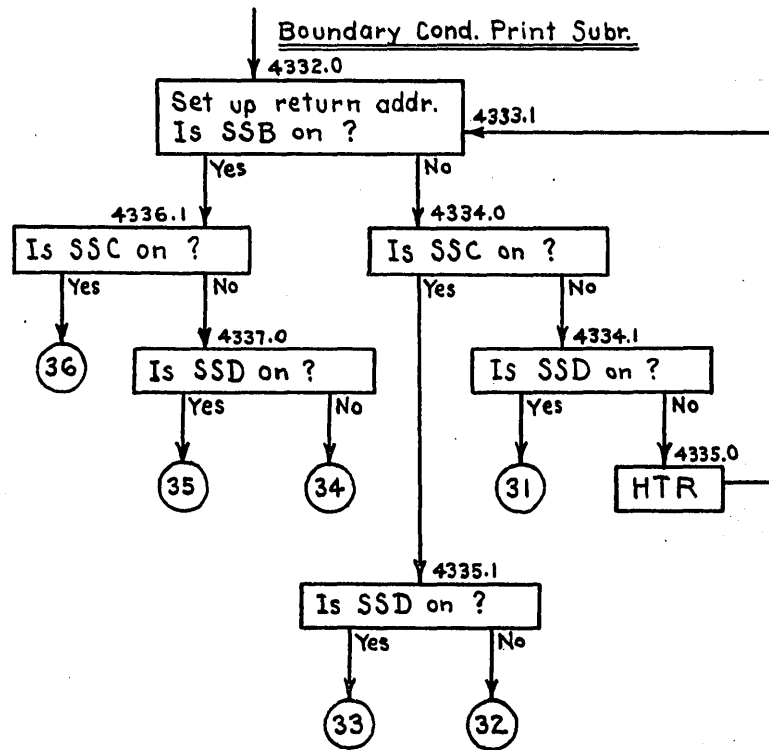
The data input and computer printout is shown on page 16 to 22. Note that since the ratio of inside to outside diameter, $\frac{d}{D}$, is 0.5, $\text{Eff.}D = 1.118 \times 0.25 = 0.28$.

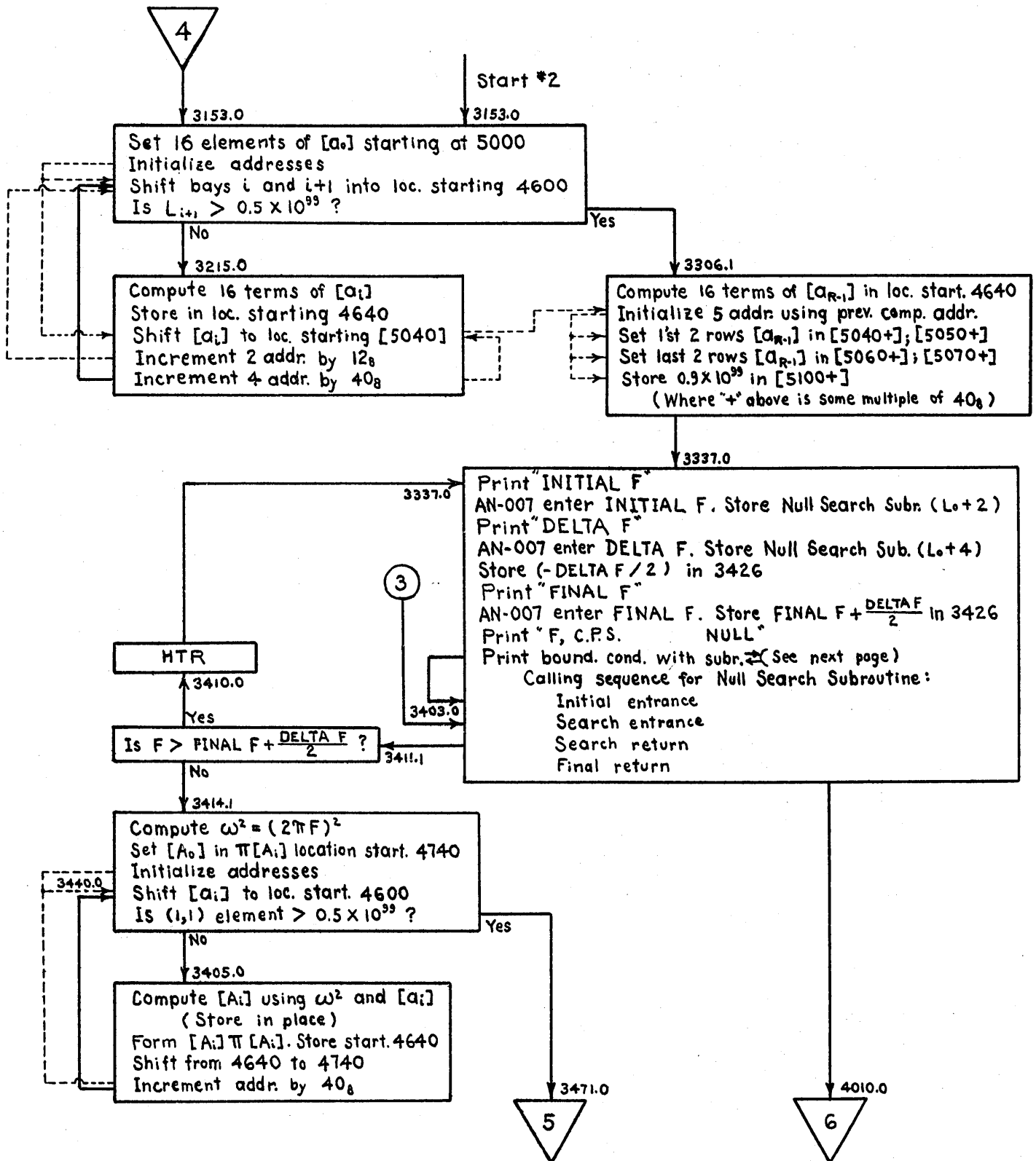
E 30+6
G 12+6

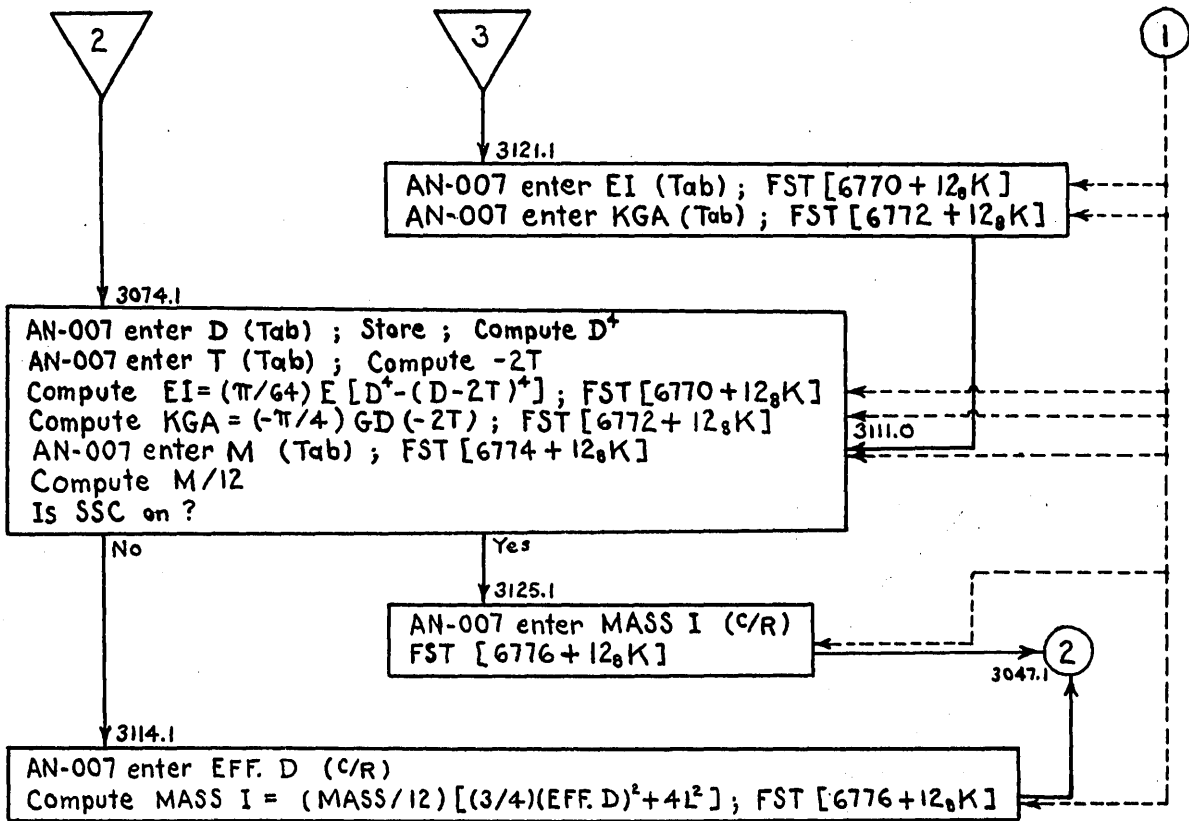
BAY	L	D	T	MASS	EFF. D
1	U	0.25	0.0625	1335-7	0.20
2	U	0.25	0.0625	1335-7	0.20
3	U	0.25	0.0625	1335-7	0.20
4	U	0.25	0.0625	1335-7	0.20
5	U	0.25	0.0625	1335-7	0.20
6	U	0.25	0.0625	1335-7	0.20
7	U	0.25	0.0625	1335-7	0.20
8	U	0.25	0.0625	1335-7	0.20
9	U	0.25	0.0625	1335-7	0.20
10	U	0.25	0.0625	1335-7	0.20
99					

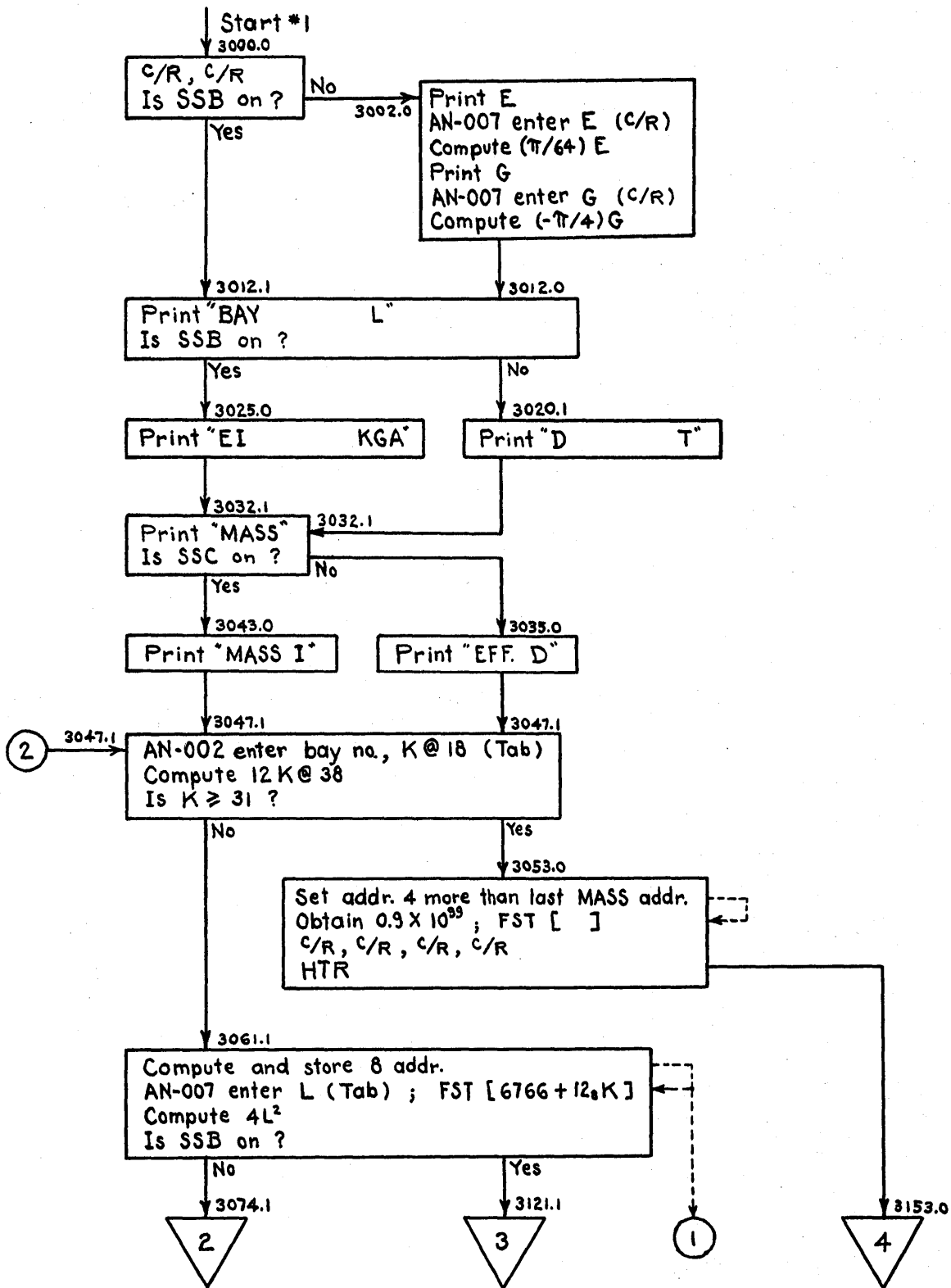












F, C.P.S.

NULL

CHECK

FREE-FREE

+ .104658 + 3

- .000000 + 0

- .306329 +12

STA. X

S

M

H

THETA

0	+ .0000 + 0	+ .000000 + 0	+ .000000 + 0	+ .100000 + 1	- .101491 + 0
1	+ .0000 + 0	+ .106492 + 5	+ .203311 + 5	+ .100000 + 1	- .101491 + 0
2	+ .1000 + 2	+ .182048 + 5	- .191191 + 6	- .152211 - 1	- .101162 + 0
3	+ .1900 + 2	+ .718873 + 4	- .374203 + 6	- .621937 + 0	- .132877 - 1
4	+ .2800 + 2	- .946852 + 4	- .442081 + 6	- .649776 + 0	+ .103431 - 1
5	+ .3700 + 2	- .267293 + 5	- .341190 + 6	- .509201 + 0	+ .182052 - 1
6	+ .4600 + 2	- .416041 + 5	- .668653 + 5	- .311344 + 0	+ .201705 - 1
7	+ .5200 + 2	- .518417 + 5	+ .214289 + 6	- .182118 + 0	+ .200314 - 1
8	+ .5800 + 2	- .548689 + 5	+ .555472 + 6	- .538505 - 1	+ .191438 - 1
9	+ .6400 + 2	- .508332 + 5	+ .913587 + 6	+ .671968 - 1	+ .174157 - 1
10	+ .7000 + 2	- .407565 + 5	+ .124244 + 7	+ .174313 + 0	+ .152835 - 1
11	+ .7600 + 2	- .258803 + 5	+ .150588 + 7	+ .264633 + 0	+ .120082 - 1
12	+ .8200 + 2	- .729342 + 4	+ .167408 + 7	+ .330643 + 0	+ .820774 - 2
13	+ .8800 + 2	+ .142854 + 5	+ .172700 + 7	+ .369190 + 0	+ .413743 - 2
14	+ .9400 + 2	+ .365438 + 5	+ .164007 + 7	+ .381375 + 0	+ .769152 - 3
15	+ .1000 + 3	+ .571969 + 5	+ .141623 + 7	+ .367399 + 0	- .290390 - 2
16	+ .1060 + 3	+ .757006 + 5	+ .106378 + 7	+ .329163 + 0	- .589104 - 2
17	+ .1120 + 3	+ .866149 + 5	+ .587777 + 6	+ .272108 + 0	- .789907 - 2
18	+ .1200 + 3	+ .906538 + 5	- .115930 + 6	+ .186803 + 0	- .818478 - 2
19	+ .1280 + 3	+ .930438 + 5	- .853998 + 6	+ .103320 + 0	- .777243 - 2
20	+ .1360 + 3	+ .940052 + 5	- .165382 + 7	+ .260411 - 1	- .697971 - 2
21	+ .1430 + 3	+ .809087 + 5	- .241279 + 7	- .403822 - 1	- .569331 - 2
22	+ .1500 + 3	+ .507128 + 5	- .304908 + 7	- .931070 - 1	- .394425 - 2
23	+ .1570 + 3	+ .100779 + 5	- .343689 + 7	- .125295 + 0	- .185095 - 2
24	+ .1640 + 3	- .329694 + 5	- .350031 + 7	- .132733 + 0	+ .401677 - 3
25	+ .1710 + 3	- .701020 + 5	- .322347 + 7	- .114496 + 0	+ .259770 - 2
26	+ .1780 + 3	- .938071 + 5	- .265244 + 7	- .730931 - 1	+ .452981 - 2
27	+ .1850 + 3	- .983754 + 5	- .188875 + 7	- .140859 - 1	+ .603762 - 2
28	+ .1920 + 3	- .806101 + 5	- .107532 + 7	+ .547780 - 1	+ .703961 - 2
29	+ .1990 + 3	- .401423 + 5	- .377119 + 6	+ .124780 + 0	+ .755420 - 2
30	+ .2060 + 3	- .722542 + 4	- .689372 + 5	+ .187620 + 0	+ .770771 - 2
31	+ .2260 + 3	+ .262008 - 2	- .471728 - 1	+ .355198 + 0	+ .767086 - 2

INITIAL F: 104.0
 DELTA F: 0.2
 FINAL F: 105.0

F, C.P.S.

NULL

FREE-FREE

+ .10400000000	+ 3	+ .14788130746	+16
+ .10420000000	+ 3	+ .10408402270	+16
+ .10440000000	+ 3	+ .59281178729	+15
+ .10460000000	+ 3	+ .13467406828	+15
+ .10480000000	+ 3	- .33365775562	+15
+ .10465751224	+ 3	+ .10447507948	+13
+ .10465795701	+ 3	+ .85899345920	+10
+ .10465796067	+ 3	- .21474836480	+10
+ .10465795993	+ 3	- .10737418240	+10
+ .10465795961	+ 3	+ .53687091200	+10
+ .10465795988	+ 3	- .00000000000	+ 0

INITIAL F 61
DELTA F 0.2
FINAL F 63

F, C.P.S.

NULL

FREE-FREE

+.61000000000	+ 2	-.65305799819	+14
+.61200000000	+ 2	-.36462795948	+14
+.61400000000	+ 2	-.66473510830	+13
+.61600000000	+ 2	+.24152614371	+14
+.61443164666	+ 2	-.83735085057	+11
+.61443706522	+ 2	-.10234101760	+10
+.61443713144	+ 2	-.33554432000	+ 8
+.61443713361	+ 2	-.00000000000	+ 0

F, C.P.S.

NULL

CHECK

FREE-FREE

+ .311921 + 2

- .000000 + 0

- .544833 +10

STA. X

S

M

H

THETA

0	+ .0000	+ 0	+ .000000	+ 0	+ .000000	+ 0	+ .100000	+ 1	- .277323	- 1
1	+ .0000	+ 0	+ .165422	+ 4	+ .747220	+ 4	+ .100000	+ 1	- .277323	- 1
2	+ .1000	+ 2	+ .365220	+ 4	- .167201	+ 5	+ .722414	+ 0	- .277243	- 1
3	+ .1900	+ 2	+ .439478	+ 4	- .495272	+ 5	+ .502505	+ 0	- .170566	- 1
4	+ .2800	+ 2	+ .519779	+ 4	- .895201	+ 5	+ .358204	+ 0	- .130283	- 1
5	+ .3700	+ 2	+ .592982	+ 4	- .137227	+ 6	+ .244303	+ 0	- .108061	- 1
6	+ .4600	+ 2	+ .655795	+ 4	- .191986	+ 6	+ .148027	+ 0	- .934790	- 2
7	+ .5200	+ 2	+ .701786	+ 4	- .232570	+ 6	+ .921047	- 1	- .883992	- 2
8	+ .5800	+ 2	+ .721478	+ 4	- .275828	+ 6	+ .394372	- 1	- .823122	- 2
9	+ .6400	+ 2	+ .716221	+ 4	- .320213	+ 6	- .930326	- 2	- .751729	- 2
10	+ .7000	+ 2	+ .688807	+ 4	- .364148	+ 6	- .536276	- 1	- .683389	- 2
11	+ .7600	+ 2	+ .642225	+ 4	- .406303	+ 6	- .932875	- 1	- .591034	- 2
12	+ .8200	+ 2	+ .578801	+ 4	- .445520	+ 6	- .127016	+ 0	- .488897	- 2
13	+ .8800	+ 2	+ .498616	+ 4	- .480885	+ 6	- .154216	+ 0	- .377805	- 2
14	+ .9400	+ 2	+ .408138	+ 4	- .511106	+ 6	- .174794	+ 0	- .278637	- 2
15	+ .1000	+ 3	+ .313968	+ 4	- .535808	+ 6	- .188590	+ 0	- .153033	- 2
16	+ .1060	+ 3	+ .216849	+ 4	- .554677	+ 6	- .194497	+ 0	- .221781	- 3
17	+ .1120	+ 3	+ .149983	+ 4	- .566852	+ 6	- .192236	+ 0	+ .112506	- 2
18	+ .1200	+ 3	+ .115243	+ 4	- .578640	+ 6	- .180886	+ 0	+ .180329	- 2
19	+ .1280	+ 3	+ .806266	+ 3	- .587624	+ 6	- .164699	+ 0	+ .230586	- 2
20	+ .1360	+ 3	- .132446	+ 4	- .598354	+ 6	- .144883	+ 0	+ .268784	- 2
21	+ .1430	+ 3	- .490842	+ 4	- .584243	+ 6	- .124409	+ 0	+ .307302	- 2
22	+ .1500	+ 3	- .780240	+ 4	- .544465	+ 6	- .100458	+ 0	+ .344092	- 2
23	+ .1570	+ 3	- .991591	+ 4	- .483901	+ 6	- .733657	- 1	+ .377643	- 2
24	+ .1640	+ 3	- .111714	+ 5	- .408084	+ 6	- .435829	- 1	+ .406786	- 2
25	+ .1710	+ 3	- .115070	+ 5	- .323100	+ 6	- .116476	- 1	+ .430724	- 2
26	+ .1780	+ 3	- .108776	+ 5	- .235479	+ 6	+ .218466	- 1	+ .449073	- 2
27	+ .1850	+ 3	- .925622	+ 4	- .152062	+ 6	+ .562834	- 1	+ .461880	- 2
28	+ .1920	+ 3	- .663299	+ 4	- .798725	+ 5	+ .910597	- 1	+ .469644	- 2
29	+ .1990	+ 3	- .301413	+ 4	- .259876	+ 5	+ .125620	+ 0	+ .473319	- 2
30	+ .2060	+ 3	- .478998	+ 3	- .460787	+ 4	+ .159495	+ 0	+ .474321	- 2
31	+ .2260	+ 3	- .300705	- 4	+ .222854	- 2	+ .255254	+ 0	+ .474119	- 2

INITIAL F	30.8
DELTA F	0.2
FINAL F	32.0

F, C.P.S.

NULL

FREE-FREE

+ .30800000000	+ 2	+ .41130164224	+ 13
+ .31000000000	+ 2	+ .20593853071	+ 13
+ .31200000000	+ 2	- .86149955585	+ 11
+ .31191969374	+ 2	+ .17884774400	+ 10
+ .31192132700	+ 2	+ .15728640000	+ 7
+ .31192132843	+ 2	+ .13107200000	+ 6
+ .31192132855	+ 2	- .39321600000	+ 6
+ .31192132846	+ 2	- .13107200000	+ 6
+ .31192132844	+ 2	- .13107200000	+ 6
+ .31192132844	+ 2	- .13107200000	+ 6
+ .31192132843	+ 2	- .00000000000	+ 0

E 30+6
G 12+6

BAY	L	D	T	MASS	EFF. D
17	8	18	0.1	0.05	22
18	8	20	0.1	0.05	28
19	8	22	0.1	0.06	30

E 40+6
G 16+6

BAY	L	D	T	MASS	MASS I
20	7	24	0.05	0.75	41
21	7	24	0.05	0.75	41
22	7	24	0.05	0.75	41
23	7	24	0.05	0.75	41
24	7	24	0.05	0.75	41
25	7	24	0.05	0.75	41
26	7	24	0.05	0.75	41
27	7	24	0.05	0.75	41
28	7	24	0.05	0.75	41
29	7	24	0.05	0.75	41

BAY	L	EI	KGA	MASS	MASS I
30 99	20	1.8+9	1.3+7	0.12	10

BAY	L	EI	KGA	MASS	MASS I
1	10	1+9	1+8	0.1	4

E 12+6
G 5+6

BAY	L	D	T	MASS	EFF. D
2	9	4	0.1	0.03	5
3	9	7	0.1	0.05	9
4	9	10	0.1	0.07	13
5	9	13	0.1	0.09	16

BAY	L	EI	KGA	MASS	EFF. D
6	6	2.5+9	3+7	0.13	16
7	6	2.5+9	3+7	0.13	16
8	6	2.5+9	3+7	0.13	16
9	6	3+9	3.5+7	0.14	16
10	6	2.5+9	3+7	0.13	16
11	6	2.5+9	3+7	0.13	16
12	6	2.5+9	3+7	0.13	16
13	6	3+9	3.5+7	0.14	16
14	6	2.5+9	3+7	0.13	16
15	6	2.5+9	3+7	0.13	16
16	6	2.5+9	3+7	0.13	16

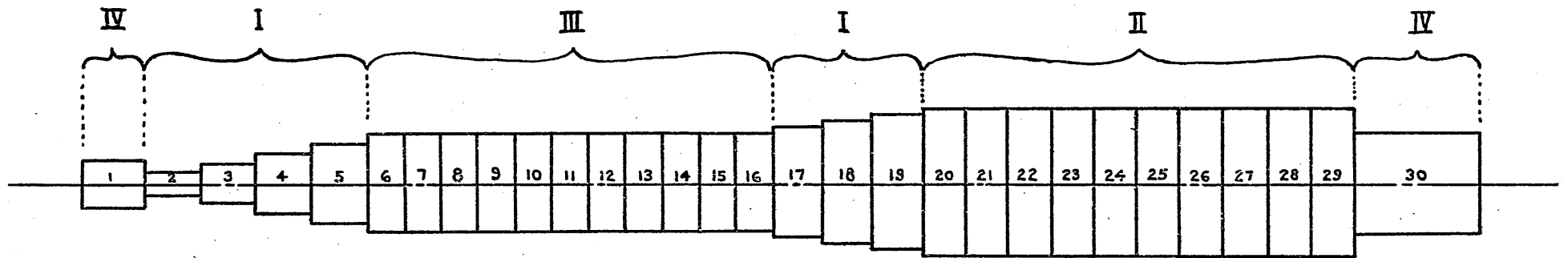


Fig. 4

EXAMPLE 2

Given a non-uniform beam as shown in the 30 bay representation shown in Fig. 4, page 24. Compute the first 3 FREE-FREE modes. This example represents and entirely fictitious multi-stage missile, and is introduced to illustrate all 4 possible data input formats with a maximum sized case.

The data input and computer printout is shown on pages 24 to 32.

INITIAL F 19.81
 DELTA F 0.02
 FINAL F 19.99

F, C.P.S.	NULL	FREE-FREE
+ .19810000000 + 2	+ .18688413239 + 3	
+ .19830000000 + 2	+ .49129325867 + 2	
+ .19850000000 + 2	- .89726325989 + 2	
+ .19837076316 + 2	+ .12605285645 + 0	
+ .19837094447 + 2	+ .33569335938 - 3	
+ .19837094495 + 2	+ .76293945313 - 5	
+ .19837094496 + 2	- .15258789063 - 4	
+ .19837094495 + 2	+ .76293945313 - 5	
+ .19837094495 + 2	- .76293945313 - 5	
+ .19837094495 + 2	- .76293945313 - 5	

F, C.P.S.	NULL	CHECK	FREE-FREE
+ .198371 + 2	- .762939 - 5	- .129856 + 5	

STA.	X	S	M	H	THETA
0	+ .0000 + 0	+ .000000 + 0	+ .000000 + 0	+ .100000 + 1	- .902124 - 1
1	+ .0000 + 0	+ .803103 + 0	+ .181241 + 1	+ .100000 + 1	- .902124 - 1
2	+ .5000 + 1	+ .193926 + 1	- .376002 + 1	+ .547826 + 0	- .900313 - 1
3	+ .1000 + 2	+ .217537 + 1	- .148752 + 2	+ .113843 + 0	- .820503 - 1
4	+ .1500 + 2	+ .164949 + 1	- .268453 + 2	- .253563 + 0	- .632166 - 1
5	+ .2000 + 2	+ .610286 + 0	- .356894 + 2	- .501078 + 0	- .345038 - 1
6	+ .2500 + 2	- .610286 + 0	- .387408 + 2	- .588526 + 0	- .909495 - 11
7	+ .3000 + 2	- .164949 + 1	- .350927 + 2	- .501078 + 0	+ .345038 - 1
8	+ .3500 + 2	- .217537 + 1	- .257521 + 2	- .253563 + 0	+ .632166 - 1
9	+ .4000 + 2	- .193926 + 1	- .134563 + 2	+ .113843 + 0	+ .820503 - 1
10	+ .4500 + 2	- .803103 + 0	- .220311 + 1	+ .547826 + 0	+ .900313 - 1
11	+ .5000 + 2	- .178807 - 8	+ .107902 - 7	+ .100000 + 1	+ .902124 - 1

INITIAL F 13.6
 DELTA F 0.1
 FINAL F 20.0

F, C.P.S. NULL

+ .13600000000 + 2	- .88068269253 + 2
+ .13700000000 + 2	+ .11826424122 + 2
+ .13688161109 + 2	- .16452503204 + 0
+ .13688323547 + 2	- .29993057251 - 3
+ .13688323843 + 2	- .95367431641 - 6
+ .13688323844 + 2	- .00000000000 + 0

F, C.P.S. NULL CHECK PIN-FREE

+ .136883 + 2 - .000000 + 0 + .146839 + 3

STA.	X	S	M	H	THETA
0	+ .0000 + 0	+ .000000 + 0	+ .000000 + 0	+ .100000 + 1	- .769272 - 1
1	+ .0000 + 0	+ .398797 + 0	+ .917675 + 0	+ .100000 + 1	- .769272 - 1
2	+ .5000 + 1	+ .100589 + 1	- .170913 + 1	+ .614771 + 0	- .768537 - 1
3	+ .1000 + 2	+ .124125 + 1	- .733915 + 1	+ .238333 + 0	- .729375 - 1
4	+ .1500 + 2	+ .113798 + 1	- .140662 + 2	- .104570 + 0	- .632560 - 1
5	+ .2000 + 2	+ .758907 + 0	- .201479 + 2	- .383870 + 0	- .475769 - 1
6	+ .2500 + 2	+ .193917 + 0	- .241659 + 2	- .572135 + 0	- .271379 - 1
7	+ .3000 + 2	- .449017 + 0	- .251707 + 2	- .651066 + 0	- .428312 - 2
8	+ .3500 + 2	- .105719 + 1	- .227773 + 2	- .615866 + 0	+ .180130 - 1
9	+ .4000 + 2	- .152830 + 1	- .171894 + 2	- .477072 + 0	+ .366805 - 1
10	+ .4500 + 2	- .178477 + 1	- .914374 + 1	- .259705 + 0	+ .490751 - 1
11	+ .5000 + 2	- .185070 + 1	+ .721229 - 9	- .181899 - 11	+ .534159 - 1

INITIAL F 8.7
 DELTA F 0.1
 FINAL F 10.0

F, C.P.S.	NULL	PIN-PIN
+ .870000000000 + 1	- .43935381532 + 2	
+ .880000000000 + 1	+ .72675204277 + 1	
+ .87858064286 + 1	- .20602464676 - 1	
+ .87858465517 + 1	- .94771385193 - 5	
+ .87858465702 + 1	- .000000000000 + 0	

F, C.P.S.	NULL	CHECK	PIN-PIN
+ .878585 + 1	- .000000 + 0	+ .887585 + 2	

STA.	X	S	M	H	THETA
0	+ .0000 + 0	+ .100000 + 1	+ .000000 + 0	+ .000000 + 0	- .477421 - 1
1	+ .0000 + 0	+ .975722 + 0	- .809755 - 1	- .000000 + 0	- .477421 - 1
2	+ .5000 + 1	+ .880211 + 0	- .511361 + 1	- .234770 + 0	- .454054 - 1
3	+ .1000 + 2	+ .698540 + 0	- .964569 + 1	- .446559 + 0	- .386242 - 1
4	+ .1500 + 2	+ .448490 + 0	- .132336 + 2	- .614636 + 0	- .280621 - 1
5	+ .2000 + 2	+ .154539 + 0	- .155261 + 2	- .722548 + 0	- .147531 - 1
6	+ .2500 + 2	- .154539 + 0	- .162988 + 2	- .759732 + 0	- .153477 - 1
7	+ .3000 + 2	- .448490 + 0	- .154760 + 2	- .722548 + 0	+ .147531 - 1
8	+ .3500 + 2	- .698540 + 0	- .131384 + 2	- .614636 + 0	+ .280621 - 1
9	+ .4000 + 2	- .880211 + 0	- .951466 + 1	- .446559 + 0	+ .386242 - 1
10	+ .4500 + 2	- .975722 + 0	- .495958 + 1	- .234770 + 0	+ .454054 - 1
11	+ .5000 + 2	- .100000 + 1	+ .654836 - 10	- .818545 - 10	+ .477421 - 1

INITIAL F 3.1
 DELTA F 0.1
 FINAL F 5.0

F, C.P.S.	NULL	FIX-FREE
+ .31000000000 + 1	+ .51369666136 - 1	
+ .32000000000 + 1	- .87549471791 - 2	
+ .31854386636 + 1	+ .10284227028 - 3	
+ .31856077262 + 1	+ .20033621695 - 6	
+ .31856080555 + 1	+ .33469405025 - 9	
+ .31856080561 + 1	+ .10186340660 - 9	
+ .31856080562 + 1	+ .14551915228 -10	
+ .31856080563 + 1	- .14551915228 -10	

F, C.P.S.	NULL	CHECK	FIX-FREE
+ .318561 + 1	- .145519 -10	+ .150897 + 1	

STA.	X	S	M	H	THETA
0	+ .0000 + 0	+ .000000 + 0	+ .000000 + 0	+ .100000 + 1	- .274330 - 1
1	+ .0000 + 0	+ .249081 - 1	+ .607382 - 1	+ .100000 + 1	- .274330 - 1
2	+ .5000 + 1	+ .710537 - 1	- .760356 - 1	+ .862790 + 0	- .274315 - 1
3	+ .1000 + 2	+ .109888 + 0	- .443433 + 0	+ .726082 + 0	- .271964 - 1
4	+ .1500 + 2	+ .141526 + 0	- .100470 + 1	+ .591551 + 0	- .265305 - 1
5	+ .2000 + 2	+ .166224 + 0	- .172360 + 1	+ .461771 + 0	- .252710 - 1
6	+ .2500 + 2	+ .184411 + 0	- .256511 + 1	+ .340051 + 0	- .232877 - 1
7	+ .3000 + 2	+ .196727 + 0	- .349630 + 1	+ .230267 + 0	- .204820 - 1
8	+ .3500 + 2	+ .204039 + 0	- .448742 + 1	+ .136718 + 0	- .167844 - 1
9	+ .4000 + 2	+ .207461 + 0	- .551303 + 1	+ .639816 - 1	- .121510 - 1
10	+ .4500 + 2	+ .208360 + 0	- .655326 + 1	+ .168029 - 1	- .655877 - 2
11	+ .5000 + 2	+ .208360 + 0	- .759506 + 1	+ .195541 -10	+ .255795 -12

INITIAL F 13.6
 DELTA F 0.1
 FINAL F 20.0

F, C.P.S.	NULL	FIX-PIN
+ .13600000000 + 2	- .83280417137 - 1	
+ .13700000000 + 2	+ .11616626289 - 1	
+ .13687758706 + 2	- .10605435818 - 4	
+ .13687769871 + 2	- .13969838619 - 8	
+ .13687769873 + 2	+ .46566128730 - 9	
+ .13687769872 + 2	+ .00000000000 + 0	

F, C.P.S.	NULL	CHECK	FIX-PIN
+ .136878 + 2	+ .000000 + 0	+ .112960 + 2	

STA.	X	S	M	H	THETA
0	+ .0000 + 0	+ .100000 + 1	+ .000000 + 0	+ .000000 + 0	- .325436 - 1
1	+ .0000 + 0	+ .959832 + 0	- .133973 + 0	- .000000 + 0	- .325436 - 1
2	+ .5000 + 1	+ .003111 + 0	- .518174 + 1	- .158716 + 0	- .301946 - 1
3	+ .1000 + 2	+ .512225 + 0	- .939102 + 1	- .294589 + 0	- .235289 - 1
4	+ .1500 + 2	+ .128511 + 0	- .120644 + 2	- .388497 + 0	- .136348 - 1
5	+ .2000 + 2	- .294219 + 0	- .127252 + 2	- .428212 + 0	- .215125 - 2
6	+ .2500 + 2	- .699663 + 0	- .111803 + 2	- .410605 + 0	+ .896488 - 2
7	+ .3000 + 2	- .103791 + 1	- .753615 + 1	- .342557 + 0	+ .177069 - 1
8	+ .3500 + 2	- .127543 + 1	- .216305 + 1	- .240537 + 0	+ .222902 - 1
9	+ .4000 + 2	- .140278 + 1	+ .438979 + 1	- .128970 + 0	+ .213394 - 1
10	+ .4500 + 2	- .144016 + 1	+ .115191 + 2	- .378512 - 1	+ .140100 - 1
11	+ .5000 + 2	- .144016 + 1	+ .187199 + 2	- .432010 - 10	- .227374 - 12

INITIAL F 19.6
 DELTA F 0.1
 FINAL F 25.0

F, C.P.S.	NULL	FIX-FIX
+ .19600000000 + 2	+ .32686996565 - 3	
+ .19700000000 + 2	+ .18786074907 - 3	
+ .19800000000 + 2	+ .48913952924 - 4	
+ .19900000000 + 2	- .89960592959 - 4	
+ .19835221683 + 2	- .87457010523 - 8	
+ .19835215387 + 2	- .18189894036 -11	
+ .19835215386 + 2	+ .18189894036 -11	
+ .19835215386 + 2	- .18189894036 -11	
+ .19835215386 + 2	- .00000000000 + 0	

F, C.P.S.	NULL	CHECK	FIX-FIX
+ .198352 + 2	- .000000 + 0	- .472896 - 1	

STA.	X	S	M	H	THETA
0	+ .0000 + 0	+ .100000 + 1	+ .110842 + 2	+ .000000 + 0	+ .000000 + 0
1	+ .0000 + 0	+ .100000 + 1	+ .110842 + 2	+ .000000 + 0	+ .000000 + 0
2	+ .5000 + 1	+ .954702 + 0	+ .594659 + 1	- .218455 - 1	- .795880 - 2
3	+ .1000 + 2	+ .805923 + 0	+ .978413 + 0	- .717510 - 1	- .112593 - 1
4	+ .1500 + 2	+ .542135 + 0	- .322926 + 1	- .127216 + 0	- .102984 - 1
5	+ .2000 + 2	+ .191420 + 0	- .604450 + 1	- .169137 + 0	- .604781 - 2
6	+ .2500 + 2	- .191420 + 0	- .700160 + 1	- .184630 + 0	- .682121 -12
7	+ .3000 + 2	- .542135 + 0	- .593993 + 1	- .169137 + 0	+ .604781 - 2
8	+ .3500 + 2	- .805923 + 0	- .305120 + 1	- .127216 + 0	+ .102984 - 1
9	+ .4000 + 2	- .954702 + 0	+ .117308 + 1	- .717510 - 1	+ .112593 - 1
10	+ .4500 + 2	- .100000 + 1	+ .608420 + 1	- .218455 - 1	+ .795880 - 2
11	+ .5000 + 2	- .100000 + 1	+ .110842 + 2	- .732143 -10	- .665068 -11